Comparison of the Lower Bound for the Capacity of the Wireless Ad Hoc Networks with Cooperating Nodes to the Upper Bound of Conventional Peer-to-Peer Wireless Networks with Hop-by-Hop Routing

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The open literature contains numerous studies of capacity bounds for an N-node wireless ad-hoc network but the number of cooperating nodes has generally been limited to two. In this paper, we present a simple time-share lower bound for such a network, but with two or more cooperating nodes. These results are then compared against our previous information theoretic upper bounds with hop-by-hop routing, frequency reuse, and no cooperation. While simulations show that our simple time-share lower bound with cooperation does not outperform our previous upper bound without cooperation, we do note a significant increase in achievable capacity relative to time-share results without cooperation. Also, our results indicate that there is no significant improvement when more than two nodes cooperate.

Keywords: Ad Hoc Networks, Capacity Bounds, Cooperation, Spatial Diversity, Wireless Ad Hoc Networks.

1 INTRODUCTION

Node cooperation in wireless ad-hoc networks has enjoyed a tremendous amount of attention in the past decade. There is an abundance of papers wherein two nodes cooperatively transmit to a single node [1–4] or two nodes cooperatively transmit or receive [3,5,6]. However, there are very few papers wherein more than two nodes cooperate [7,8], and we have not come across
any papers where consideration is placed on comparing the benefits of having more than two nodes cooperating. In this paper, we explore a simple time-share lower bound on the capacity of wireless ad-hoc networks with two or more cooperating nodes, and comparing this lower bound with our previous information theoretic upper bounds [9,10] for wireless ad-hoc networks with no cooperation.

In our earlier work, we determined the maximum factor that can be applied to scale some given traffic matrix such that if each element of the traffic matrix is scaled by a greater value, then it would become impossible for the network to deliver all the packets. Our intent was to use this upper bound as a yardstick against future results based on cooperation.

The goal of this study is to determine if there is any benefit in adding node cooperation to ad-hoc networks including an assessment of whether there is any benefit in allowing more than two nodes to cooperate. In our model, two or more nodes may form a group by receiving a common transmission sent by a single node, and can then cooperate to forward this transmission to some other single node. Thus, in our model, a group of nodes does not cooperate to send a message to another group of nodes. We obtained our results with cooperation by using a time-share strategy (no frequency re-use) thereby producing a lower bound on network capacity, and compare this to our frequency reuse upper bounds (hop-by-hop routing without cooperation).

Figure 1 depicts the type of ideal curves one might achieve with our method. However, we actually expect our results with cooperation to be lower than our previous upper bounds without cooperation because no frequency re-use is allowed. If our current results are close to the upper bounds, it would be worthwhile to explore further. On the other hand, if our bounds are much worse than the upper bounds, then either our upper bounds are too loose, or we need to incorporate a more sophisticated algorithm beside our simple time-share method.

In the next section, we will summarize related work done by other researchers. In Section 3, we present our network model and our initial parameters. Section 4 is dedicated in presenting how the optimal path is determined. Finally, we present our results in section 5 and conclude our paper in section 6.

![FIGURE 1](image)
Ideal results for nodes cooperation.
It is found that for a simple time-share method, we could not outperform our previous upper bounds with frequency reuse. However, based strictly on our time-share lower bound, we do note a significant improvement when cooperation is used as compared against simple time-shared hop-by-hop routing. Also, our results suggest there is little additional benefit when the number of cooperating nodes exceeds two. We also observed that as SNR increases, the benefits of using node cooperation decreases. This is perhaps because at the higher SNR, we can better exploit the natural diversity of an ad-hoc network to overcome the effects of shadow and multipath fading.

2 RELATED WORK

Sending directly from the source node to the destination node requires a large amount of power to transmit over a long distance. Researchers have been exploring spatial diversity to reduce the amount of power necessary for transmission. One obvious way to introduce spatial diversity is to increase the number of antennas on a device for reception. However, multiple antennas on a mobile device such as a cell phone is not desirable. Unlike a wired network, when a node sends a packet to another node in a wireless medium, all the other nodes within the hearing range will be able to receive the packet. Rather than discarding the packets they received, they may cooperatively send to the destination node using a lower power level.

Node cooperation is not a new topic. Cover et al. [11] may as well be one of the first researchers to suggest the potential benefits of node cooperation. Using cooperative nodes in transmitting information allows more even use of the network, thus allowing a more distributed use of power. The downside of using the single hop-by-hop method without cooperation is that it could cause a bottleneck or a fast draining of power among the popular nodes. Sendonaris, Goldsmith, Stanković and El Gamal referenced earlier in this paper are just a few researchers who have extensively analyzed the benefits of adding cooperation into the network.

Recently, researchers have broadened the area from looking at the complexity or benefits of node cooperation to determining the trade off between node cooperation and interference under the decode and forward scheme [12]. Another group of researchers used the cooperation decision parameter (CDP), a function of user-to-destination average received SNR, to determine if cooperation would be beneficial or not without consideration for inter user link quality [13].

3 NETWORK MODEL

Consider a network with N nodes each with a transmitter of P watts. We define the spacing between the N nodes as randomly distributed over a grid. We use
the same setup as with our previous paper [14]. First, we normalize the SNR by considering a square grid with length $L$ and with nodes equally spaced $d$ distance apart. There will be $\sqrt{N}$ nodes in each row and column. $d$ is defined as

$$d = \frac{L}{\sqrt{N} - 1}$$  \hspace{1cm} (1)

If the transmitting and receiving nodes are of distance $d$ apart, we can determine the normalized SNR with no interference between the two nodes. We then randomly distribute $N$ nodes over the grid, if the two nodes are not spaced distance $d$ apart, we would scale the SNR appropriately.

We further define the traffic flow between each node to create an arbitrary $N$ by $N$ traffic matrix: traffic going between node $i$ to node $j$ is matrix element $t_{ij}$. The traffic matrix for all the nodes is then

$$\bar{T} = \begin{bmatrix} t_{11} & t_{12} & \cdots & t_{1N} \\ t_{21} & t_{22} & \cdots & t_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ t_{N1} & t_{N2} & \cdots & t_{NN} \end{bmatrix}$$  \hspace{1cm} (2)

Our traffic matrix represents the exogenous traffic between two nodes and is in units of fixed length packets per second. The traffic matrix does not need to be symmetric, because the two nodes may be of different equipment and their sending rate could be different. We use this matrix to defines the traffic that the network must deliver.

We next define our capacity matrix. This interference free capacity matrix represents point-to-point Shannon capacity. The capacity matrix is calculated using Shannon Capacity $C = W \log(1 + \rho)$. $\rho$ is the pre-defined interference free SNR and $W$ is the bandwidth. We set $W$ to one so that our results will be in terms of bits per second per hertz. This capacity matrix is based on each node sending at its maximum power. For a simple single hop-by-hop network, the capacity matrix is

$$\bar{C} = \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1N} \\ C_{21} & C_{22} & \cdots & C_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ C_{N1} & C_{N2} & \cdots & C_{NN} \end{bmatrix}$$  \hspace{1cm} (3)

Now, when cooperation is permitted, we introduce phantom nodes into the network. When two nodes are allowed to cooperate, we define a set of phantom node pairs, each consisting of one of $\frac{N(N-1)}{2}$ possible pairs of nodes. A single node may send to a single other node or to a node pair; a node pair, however, can send only to a single node. Similarly, when three nodes are allowed to cooperate, there are $\frac{N(N-1)(N-2)}{6}$ phantom node triplets. A single node may send to a single node, a node pair, or a node triplet, but a node pair
or triplet can send only to a single node. Extensions to m-node cooperation are straightforward. Figure 2 shows a few possible paths a source node may use a single node or a set of phantom node pairs to transmit its exogenous traffic to the destination node.

There may be numerous capacity matrices for a network with node cooperation. For example, to allow paired-node cooperation, we must add a capacity matrix that represents the capacity between two nodes sending to one node. We must also consider the event where a node is sending to a two-node pair. For conventional hop-by-hop networks, we have the same capacity as stated above. In the case where we are sending to two nodes or more, we use the worst capacity link as the bottleneck, thus we take the minimum of the capacity links. For example, if node \( i \) is sending to node \( j, k \) and \( l \), we have

\[
C_{ij}^{jkl} = \min\{C_{ij}, C_{ik}, C_{il}\}
\]  

\( C_{ij}^{jkl} \) would be an element within the new capacity matrix for a single node sending to three nodes. In order to determine the capacity for each transmission easily, we have a capacity matrix for each scenario; a single node to a paired-node, a single node to three nodes, and so on.

On the other hand, if node \( j \) and node \( k \) are cooperatively sending to node \( i \), we use superposition to determine the SNR of the cooperative transmission. The SNR is defined as the signal power over the noise power and is in unit of volt\(^2\) or watts. We start with using volt\(^2\) to derive the signal power arriving at node \( i \). To determine the SNR at the receiving node, we use \( v = \sqrt{\rho} \), where \( \rho \) is the SNR in watt. The signal voltage arriving at node \( i \) from node \( j \) and node \( k \) is then

\[
v_{ij}^i = v_{ji} + v_{ki}
\]  

\[
v_{jk}^i = \sqrt{\rho_{ji}} + \sqrt{\rho_{ki}}
\]
then taking the signal voltage arriving at the receiving end and convert it to power, we have

\[(v_{jk}^i)^2 = (\sqrt{\rho_{ji} + \sqrt{\rho_{kl}}})^2.\]  \hspace{1cm} (7)

Using this new SNR value, we calculate the capacity as before. We generate a different capacity matrix for each scenario; we have a pair of nodes transmitting cooperatively to a single node, three nodes transmitting cooperatively to a single node, and so on.

So far, we have only mentioned how to calculate the capacity matrix without loss. To obtain a realistic model, we must include the channel impairments. Our model has three types of impairments: Path loss, multipath fading and shadow fading. Path loss is defined as the power attenuation as a signal propagates across a medium. Our path loss has the highest impact on the capacity of the network and is defined as

\[r = \left(\frac{d}{R}\right)^4\]  \hspace{1cm} (8)

d is from equation (1) and R is the coordinate of each node in \((x, y)\) location of the grid.

The multipath fading between two nodes is defined as an exponential distribution of a random variable \(X\) with a mean of one. Compared to the path loss, multipath fading has little effect to the SNR. The shadow fading is a log normal distribution with a mean of six decibel. The SNR with impairments can now be computed:

\[\rho_{ij} = \rho \ast \left(\frac{d}{R}\right)^4 \ast e^X \ast 10^{\frac{m}{10}}\]  \hspace{1cm} (9)

When computing the capacity matrix, we use the SNR with impairments.

4 SEARCH ALGORITHM

The path by which a node would transmit its exogenous traffic to the destination node is found via exhaustive search. Starting at an originating or root node, a path is found to a particular destination node by means of extending a tree from the root through each possible next-hop node or node pair, or node triplet, etc. The time needed to transmit to each node at depth one is found simply as the inverse of the capacity leading to from the root to each possible level one termination. We next extend the tree to depth two by an analogous process, except that no grouping of nodes can send to another grouping, that is, each grouping at depth one can send only to a single node at depth two. Of course, a single node at depth one can send to a node grouping at level two. We found
FIGURE 3
An example where a path with the minimum time (dotted-dashed line) is chosen even if more
hops are required to transmit a packet from the source node to the destination node.

the time consumed in sending a message to each node or node grouping by
adding the time taken to send a level two child from its parent to the time
taken to send from the root node to that parent. We successively extend the
depth of the tree in the above fashion until, by exhaustive search, we found the
minimum time it takes to send from the root node to the chosen destination.
This process is then repeated for each source-destination pair. Weighting the
time taken to send a packet from a given source to a given destination by the
relative amount of traffic from that source to that destination (gotten from the
traffic matrix) and summing over all source-destination pairs then tells us the
minimum amount of time it takes to send all the traffic in time-share mode.
Figure 3 shows the path (dotted-dashed line) by which a source node would use
to transmit its exogenous traffic to the destination node in the sample network.
Mathematically, the time required to transmit from the source node \( i \) to the
destination node \( j \) in a single hop is

\[
T_{\text{ref}} \propto \frac{1}{C_{ij}} \quad (10)
\]

To calculate the time it takes for a node to send to another node or paired-
node, we take the minimum transmission time. The equation we use in
choosing between a single node or a paired-node is

\[
T_{\text{hop}} = \min [T_{\text{single}}, T_{\text{pair}}]. \quad (11)
\]

The route we take is the least total time elapsed for a node to send from
the source node to one or more intermediate nodes or a pair of nodes, and the
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final transmit time to the destination node. For example, if the source node \( i \) is sending to the destination node \( j \) via node \( k \) and node \( l \) as a pair, we have

\[
t_{\text{route}} = t_{\text{hop},i} + t_{\text{hop},kl}
\]  

(12)

where \( t_{\text{hop},i} \) is the time it takes for node \( i \) to transmit data to node \( k \) and node \( l \) and \( t_{\text{hop},kl} \) is the time it takes for node \( k \) and node \( l \) to transmit data to node \( j \). The \( t_{\text{hop}} \) is calculated by equation (11). The final time it takes for a source node to send data to the destination node is the smaller of \( t_{\text{route}} \) from equation (12) and \( t_{\text{ref}} \), we denote this value as \( t_{\text{total}} \). Note that \( t_{\text{total}} \) is computed for each \( i, j \) pair.

\[
t_{\text{total}}(i, j) = \min \left[ t_{\text{ref}}, t_{\text{route}} \right].
\]  

(13)

The total capacity of the network is then the total amount of traffic sent divided by the total time taken to send it.

\[
C_{\text{network}} = \frac{\sum_{i=1}^{N} \sum_{j=1}^{N} t_{ij}}{\sum_{i=1}^{N} \sum_{j=1}^{N} t_{ij} t_{\text{total}}(i, j)}.
\]  

(14)

5 RESULTS

After the publication of this work in another paper [14], we found a factor of 2 was incorrectly introduced in the denominator of equation (14). We have fixed the error in this paper.

Previously, we have presented our capacity bounds for the case where traffic is uniformly distributed [14]. By doing so, we were able to isolate the effects of node cooperation. We presented a comparison between three types of network: a network with and without cooperation and with no frequency reuse, and a network without cooperation but includes frequency reuse. Here, we present a set of results using Poisson distributed traffic matrix with mean 10. We repeat each scenario ten times to ensure the repeatability of our model.

To generate the results, we use path loss of \( \left( \frac{1}{R} \right)^4 \), multipath fading with exponential distribution of mean one, and shadow fading with lognormal distribution with mean six. A total of ten simulations were run for each network scenario to test the results repeatability. We focus on two scenarios where the numbers of nodes in the networks are 9 and 25.

Figure 4 shows our results for a nine-node network, using a Poisson traffic matrix with mean 10 with path loss, multipath fading and shadow fading.

Figure 5 is similar to figure 4, but with 25 nodes instead of 9 nodes.

As with our results with the uniform matrix [14], as the SNR increases, the benefit of incorporating cooperation into the network decreases. This is perhaps because with higher SNR, nodes may better exploit the natural diversity
FIGURE 4
Results for up to four nodes cooperatively transmitting, based on the average of ten trials for a 9-node network, with Poisson distributed traffic matrix of mean 10, with path loss, multi-path fading and shadow fading. The figure on the right is a close up look of the results with cooperation.

FIGURE 5
Results for up to four nodes cooperatively transmitting, based on the average of ten trials for a 25-node network, with Poisson distributed traffic matrix of mean 10, with path loss, multi-path fading and shadow fading. The figure on the right is a close up look of the results with cooperation.
FIGURE 6
A comparison with our previous upper bound for a conventional peer-to-peer network with hop-by-hop routing and with frequency reuse.

of an ad-hoc network to overcome channel impairments more readily than at a lower SNR value. However, based on our routing scheme, the traditional method of hop-by-hop method could never out perform cooperative transmission.

Comparing with our previous work of hop-by-hop routing, we found that even with cooperation, our simple strategy of time-shared bounding could not out perform our previous upper bound with frequency reuse. This could be because our upper bound is too loose, or we have room for improvement beyond simply using our time-share method. Currently, our result from the conventional peer-to-peer network with hop-by-hop routing and with frequency reuse performs roughly 150 times better than our network with node cooperation and no frequency reuse at SNR equals 0 dB, 41 times better at a SNR equal to 15 dB, and 31 times better at a SNR equal to 30 dB. Figure 6 shows the comparisons discussed above.

6 CONCLUSIONS

It is obvious that our lower capacity bound using a simple time-share method could not out perform our upper capacity bound with frequency reuse with no node cooperation. This could be because our previous upper bound is too loose, or we have much room for improvement with our lower bounds. It is our intention to pursue frequency reuse with node cooperation. By introducing frequency reuse, we will also create interference in our network.
As SNR increases, the benefit of node cooperation decreases. This may be because as SNR increases, a node is more likely to be capable of overcoming the channel impairments without the help of node cooperation. Another observation worth noting is that there is no significant improvement by combining more nodes together for cooperation. Three-node cooperation has only a slight increase of capacity over paired-node cooperation.

For future improvements of our algorithm, we will start with limiting which nodes may send or receive by putting a restriction around the nodes that are receiving or transmitting. We plan to put a non-physical 3dB capacity circle around the transmitting node and the receiving node. Any nodes outside those 3dB circles may send or receive at the same time. Such method would require intensive bookkeeping.

Currently, we have incorporated only transmitter cooperation; we might include receiver cooperation in our future work as well. Papers from other researchers such as Goldsmith have indicated that receiver cooperation provides gain in improvement of capacity [3]. This might be worthwhile to peruse besides adding frequency reuse to our current model.

REFERENCES


