

Spectrum Allocation Mechanisms in Wireless Networks with Performance Guarantee

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With the advent of wireless communications technology, the demand for limited spectrum resources is increased rapidly. Efficient spectrum allocation techniques have been regarded as a key step that enables effective high-throughput ad hoc networking with efficient spectrum usage. In this work we focus on spectrum allocation mechanisms in the secondary market to mitigate the spectrum scarcity. In a spectrum trading market, we assume that secondary users will bid for the usage of spectrums in: 1) some fixed time intervals (*i.e.* Fixed interval), 2) some continuous time intervals in particular time ranges (*i.e.* Time-window), or 3) some time slices summed to a certain value within a time range (*i.e.* Time-window-slice). Our goal is to design spectrum allocation mechanisms that will maximize the social efficiency under these three possible bidding cases. As allocating the requests of secondary users optimally is an NP-hard problem, to this end, we propose a sub-optimal spectrum allocation mechanism PVG, in which a greedy allocation method is designed to maximize the social efficiency (total valuation of the allocated spectra). We prove that the PVG allocation mechanism yields an approximation factor $6 + 4\sqrt{2}$ for the Time-window-slice case, an approximation factor 8 for the Time-window case, and an approximation factor 32 for the Fixed-interval case. We then conduct an extensive simulation on a real spectrum availability data to evaluate the performance of PVG. Our results show that the social efficiency ratio of PVG is always above 70% compared with the optimal allocation mechanism in these three request cases.

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1 INTRODUCTION

In recent years, with the increasing popularity of wireless devices and applications (*e.g.* 4/5G, WiFi, ad hoc networks), the ever-increasing demand of traffic poses a great challenge in spectrum allocation and usage [3, 6]. For instance, traditional wireless ad hoc networks often operate in the ISM bands (900MHz or 2.4GHz). With the growing proliferation of wireless devices, these bands are increasingly getting congested [1]. On the other hand, the current fixed long-term and regional lease-based spectrum allocation scheme leads to significant spectrum white spaces and artificial shortage of spectrum resources. To obtain a better transmission quality, an intuitive idea is to make the ad hoc networks opportunistically work in the underutilized licensed bands [19]. Many efforts such as the Federal Communications Commission (FCC) ruling on white spaces are attempting to free the under-utilized licensed spectrums by permitting opportunistic access [17]. Therefore, spectrum allocation is the key technology that enables the ad hoc networks to use spectrum more efficiently.

In this work, we study the spectrum allocation in the secondary market in which a device from an ad hoc network will bid for the usage of certain spectrum and pay price to the holder of the spectrum. Previous studies on spectrum allocation mechanisms in the secondary spectrum market (*e.g.* [10, 13, 16, 23, 25, 27, 28, 34, 35]) mainly consider the wireless interference and spatial reuse of channels. Most of these existing methods assume that secondary users can share one channel only if they are spatial-conflict free with each other. However, under a more realistic model for ad hoc networking or general networking, a secondary user may be interested in the usage of a channel during some *specific time periods*. In another word, secondary users can share the same channel in *spatial*, *temporal*, and *spectral* domain as long as it does not cause interference with other devices in the (ad hoc) networking. So, it is reasonable to further improve the spectrum utilization by introducing time-domain reutilization.

By following this direction, some papers (*e.g.* [7, 24, 29, 31] and [36]) take the requested time durations of secondary users into consideration. Generally speaking, there exist three possible types of time duration requests. The first type, which can be called *Fixed-Interval*, is that all the secondary users request some fixed continuous time intervals (*e.g.*, requesting the usage of one channel for a fixed time interval exactly lasting from 2:00PM to 4:00PM). The second type, which we call *Time-Window*, allows each secondary user to

request a continuous time interval in a particular time range (*e.g.*, requesting the usage of one channel for a *continuous* time interval of 2 hours in the time range lasting from 2:00PM to 5:00PM). While in the third type, secondary users can request some time slices summed to a certain value within a time range, and we call this *Time-Window-Slice* (*e.g.*, requesting the usage of one channel for 2 hours in *total* with the time slices dispersed from 2:00PM to 5:00PM). Most of the current researches consider the Fixed-Interval case, which can be regarded as a special case of both the Time-Window and Time-Window-Slice cases. In fact, the requested time durations from a secondary user are not always fixed, and the Time-Window and Time-Window-Slice cases are more appropriate in practice. However, only few studies allow channels to be reused both in spatial and temporal domain even in the Fixed-interval case, and none of them give an approximation factor. Moreover, Time-Window case has already been considered in some works, but none of them give an approximation factor or allow channels to be reused in the spatial domain either. To the best of our knowledge, there have not yet been any studies about Time-Window-Slice case.

To tackle these challenges, we propose spectrum allocation mechanisms in which secondary users can bid for the usage of channels during some specific time periods. The time slots allocated to a fixed request should be supplied by one channel, and all the time request cases that introduced above are considered. A natural goal of spectrum allocation mechanisms is to maximize the *social efficiency*. The social efficiency is defined as the total “valuation” from the secondary users, who are successfully allocated in spectrum [2]. Therefore, our aim in this work is to design allocation mechanisms which maximize *social efficiency*, *i.e.* allocating spectrum to the secondary users who value it most. Notice that the value of a secondary user in an ad hoc networking could depend on the networking traffic and requirement of the user in the network. In the Time-Window-slice and Time-Window cases, we assume that secondary users can share one channel in temporal domain. In the Fixed-Interval case, we study a more general case where channels can be reused both in spatial and temporal domain.

The main contributions of this paper are as follows. We first study the optimal channel allocation problem in this paper. Unfortunately, allocating the spectrums to the set of buyers that will maximize the social efficiency is an NP-hard problem in our settings. To address this NP-hardness challenge, we propose a sub-optimal channel allocation framework, called PVG (Per-Value Greedy), with a *greedy-like* winner determination mechanism. We show that our PVG allocation mechanism has a polynomial time complexity and it achieves a social efficiency at least $1/(6 + 4\sqrt{2})$ times of the optimum in the Time-Window-Slice case, at least $1/8$ times of the optimum in the Time-Window case, and at least $1/32$ times of the optimum in the

Fixed-Interval case. The low time complexity makes PVG much more practical for large scale spectrum market. Notice that the theoretical bound on the social efficiency is pessimistic. To evaluate the practical performance of our mechanisms, we conduct extensive simulation studies using real spectrum availability data. Our simulation results show that the social-efficient performance of our sub-optimal mechanism is much better than the theoretical guarantee. The social efficiency achieved by our sub-optimal method is actually larger than 70% of the optimal for almost all our evaluations.

The rest of paper is organized as follows: Section 2 introduces preliminaries and our design targets. Sections 3 and 4 propose our algorithms design for optimal and suboptimal mechanisms. Section 5 evaluates the performance of our mechanisms. Section 6 reviews the related work and Section 7 concludes the paper.

2 PRELIMINARIES

2.1 Spectrum Allocation Model

Consider a spectrum setting where one primary user holds the usage right of M channels $\mathcal{S} = \{s_1, s_2, \dots, s_M\}$ and is willing to sublease the usage of these channels to secondary users by time intervals. There are N secondary users $\mathcal{B} = \{b_1, b_2, \dots, b_N\}$ who want to use channels for some period of time. We assume that each secondary user $b_j \in \mathcal{B}$ has one request I_j , and let \mathcal{I} be the set of requests of secondary users. Then each request $I_j \in \mathcal{I}$ can be described as $I_j = (v_j, a_j, d_j, t_j, L_j)$, where v_j is its bidding price for the usage right of channels, a_j , d_j and t_j respectively denote each request's arrival time, deadline and duration (or time length), L_j is the geographical location where b_j wants to access the channel. Note that for each request, if it is admitted, it will be served using one unique spectrum. In other words, we assume that the secondary users will not use one spectrum for a certain time duration and then switch to another spectrum later.

We define the spectrum usage conflict among requests of secondary users as *secondary conflict*. The secondary conflict can be modeled by a *conflict graph* $\mathcal{G} = (\mathcal{I}, \mathcal{E})$, where \mathcal{I} is the vertex set corresponding to the requests of secondary users, and an edge between two requests belongs to \mathcal{E} if and only if the two requests conflict with each other, *i.e.* they cannot access the same channel simultaneously. We study three possible types of time duration in this paper. In order to simplify the problem and focus on studying the temporal impact on the spectrum allocation mechanism design, we assume requests can only share one channel in temporal domain in the Time-Window-Slice and Time-Window cases, and let requests can share one channel both in temporal and spatial domain in the Fixed-Interval case.

Each channel $s_i \in \mathcal{S}$ provided by the primary user has a set of available time slots, denoted as A_i , which can be used by the secondary users. To ensure the worst case profit, we assume that the primary user has already set a reservation price η_s which is the minimum price for the usage of spectrum per-unit time.

2.2 Design Target

In this paper, we target at designing spectrum allocation mechanisms that can maximize the social efficiency. Here, social efficiency is introduced to evaluate the performance of the proposed mechanism. The social efficiency for a spectrum allocation mechanism \mathcal{M} is defined as the total bids of all winners, *i.e.* $\text{EFF}(\mathcal{M}) = \sum_{I_j \in \mathcal{I}} v_j z_j$, where z_j indicates whether the request of the secondary user I_j is satisfied or not. Then, we will concern with the following optimization problem: designing an allocation scheme (approximately) that maximizes the social efficiency $\sum_{I_j \in \mathcal{I}} v_j z_j$.

3 OPTIMAL SPECTRUM ALLOCATION

In this section, we propose an optimal allocation mechanism which can maximize the social efficiency. The basic idea is to achieve the maximization of the social efficiency through an optimal matching between the request set \mathcal{I} and the spectrum set \mathcal{S} . More specifically, we first derive the detailed optimization problems for the Time-Window-Slice, Time-Window and Fixed-interval cases, respectively. Then, we present the optimal allocation mechanism for them.

3.1 The Time-Window-Slice Allocation Model

We assume that $A_i = \{x_{1,i}, \dots, x_{q,i}\}$ includes all the available time slots in s_i . Each $I_j \in \mathcal{I}$ can only be allocated in the time slot of s_i between a_j and d_j . In order to simplify the matching model between I_j and s_i , we will make a further segmentation to A_i based on the arrival time and deadline of all the requests in \mathcal{I} . For each $I_j \in \mathcal{I}$, its arrival time/deadline divides a time slot in s_i into 2 time slots. As shown in Figure 1, the time axis of s_i is divided into many disjoint time slots after our further segmentation. Let $x_{l,i}$ be the l -th time slot in s_i and $\Delta_{l,i}$ be the length of $x_{l,i}$. We define $\Delta_{l,i} = 0$ when time slot $x_{l,i}$ is occupied by the primary user. Assume that the time slot beginning at a_j is the $n_s^{i,j}$ -th time slot in s_i and the time slot ending at d_j is the $n_e^{i,j}$ -th time slot in s_i . Formally, $z_{i,j} \in \{0, 1\}$ is a binary variable indicating whether I_j is allocated in s_i . We can formulate the spectrum assignment into an IP (*Integral Programming*) problem:

$$\max O(v) = \sum_{I_j \in \mathcal{I}} \sum_{s_i \in \mathcal{S}} v_j z_{i,j} \quad (\text{IP (1)})$$

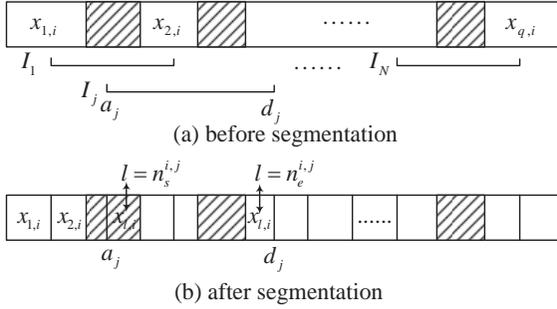


FIGURE 1

An instance of the spectrum s_i 's time axis segmentation. Let shadow slots denote the time intervals occupied by primary user, and the blank slots indicate the intervals which can be allocated to secondary users.

subject to

$$\left\{ \begin{array}{l} z_{i,j} \in \{0, 1\}, \forall s_i \in \mathcal{S}, \forall I_j \in \mathcal{I} \\ \sum_{s_i \in \mathcal{S}} z_{i,j} \leq 1, \forall I_j \in \mathcal{I} \\ \sum_{s_i \in \mathcal{S}} v_j z_{i,j} \geq \eta_s t_j z_{i,j}, \forall I_j \in \mathcal{I} \\ x_{l,i}^j \geq 0, \forall l, \forall s_i \in \mathcal{S}, \forall I_j \in \mathcal{I} \\ \sum_{l=n_s^{i,j}}^{n_e^{i,j}} x_{l,i}^j \geq t_j z_{i,j}, \forall s_i \in \mathcal{S}, \forall I_j \in \mathcal{I} \\ \sum_{I_j \in \mathcal{I}} x_{l,i}^j \leq \Delta_{l,i}, \forall s_i \in \mathcal{S}, \forall l \end{array} \right.$$

where $x_{l,i}^j$ is the time $x_{l,i}$ allocated to I_j , and $O(v)$ denotes the objective function of the IP.

3.2 The Time-Window Allocation Model

In the Time-Window case, we first simplify the allocation model by dividing the time axis for each spectrum channel into time slots of equal length. Then, we let the arrival time a_j be the first available time slot, let the deadline d_j be the last one, and let the time length t_j be the number of time slots required by request I_j . In this way, we can convert the Time-Window allocation model into the Fixed-Interval allocation model. Let $f_j = (d_j - a_j) - t_j + 1$. Then, for each request I_j , there are f_j possible allocations in each spectrum channel. Thus, we can split request I_j into f_j different requests, and all these requests conflict with each other. Assuming that F_j is the request set obtained by splitting request I_j , and $z_{i,j'} \in \{0, 1\}$ is a binary variable indicating whether $I_{j'}$ is allocated in s_i . Then, these split requests $I_{j'}$ should satisfy $\sum_{j' \in F_j} z_{i,j'} \leq 1$.

Through the request splitting above, we convert the Time-Window allocation model into the Fixed-Interval allocation model, which is formulated as an IP like this:

$$\max O(v) = \sum_{s_i \in \mathcal{S}} \sum_{I_j \in \mathcal{I}} \sum_{I_{j'} \in F_j} v_j z_{i,j'} \quad (\text{IP (2)})$$

subject to

$$\left\{ \begin{array}{l} \sum_{s_i \in \mathcal{S}} \sum_{I_{j'} \in F_j} z_{i,j'} \leq 1, \forall I_j \in \mathcal{I} \\ \sum_{l=a_{j'}}^{d_{j'}} x_{l,i}^{j'} = t_j z_{i,j'}, \forall s_i \in \mathcal{S}, \forall I_j \in \mathcal{I}, \forall I_{j'} \in F_j \\ \sum_{s_i \in \mathcal{S}} \sum_{I_{j'} \in F_j} v_j y_{i,j'} \geq \eta_s t_j y_{i,j'}, \forall I_j \in \mathcal{I} \\ z_{i,j'} \in \{0, 1\}, \forall s_i \in \mathcal{S}, \forall I_j \in \mathcal{I}, \forall I_{j'} \in F_j \\ x_{l,i}^{j'} \geq 0, \forall s_i \in \mathcal{S}, \forall I_j \in \mathcal{I}, \forall I_{j'} \in F_j, \forall l \\ \sum_{I_j \in \mathcal{I}} \sum_{I_{j'} \in F_j} x_{l,i}^{j'} \leq \Delta_{l,i}, \forall s_i \in \mathcal{S}, \forall l \end{array} \right.$$

where $x_{l,i}^{j'}$ is the time allocated to $I_{j'}$ from the l -th time slot in s_i , and $O(v)$ denotes the objective function of the IP.

3.3 The Fixed-interval Allocation Model

In the Fixed-Interval allocation case, channels can be reused both in the spatial and temporal domain. Let $y_{j,k} = 0, 1$ be a binary variable indicating whether request I_j is conflict with request I_k . Obviously, if the distance between L_j and L_k is less than twice of the interference radius and the time I_j and I_k requested are overlap with each other, $y_{j,k} = 1$; otherwise, $y_{j,k} = 0$. In this case, we first segment the available time of each channel into many time slices and use the same symbols as we did in the Time-window-slice case. Then, the optimal channel allocation problem can be formulated as follows.

$$\max O(v) = \sum_{s_i \in \mathcal{S}} \sum_{I_j \in \mathcal{I}} v_j z_{i,j}, \quad (\text{IP (3)})$$

subject to

$$\left\{ \begin{array}{l} \sum_{s_i \in \mathcal{S}} z_{i,j} \leq 1, \forall I_j \in \mathcal{I} \\ x_{l,i}^j = \Delta_{l,i} z_{i,j}, \forall s_i \in \mathcal{S}, \forall I_j \in \mathcal{I}, \forall l \\ \sum_{k \neq j} z_{i,k}^l y_{j,k} + z_{i,j}^l \leq 1, \forall s_i \in \mathcal{S}, \forall I_j \in \mathcal{I}, \forall l \\ \sum_{l=n_s^{i,j}}^{i,j} x_{l,i}^j = t_j z_{i,j}, \forall s_i \in \mathcal{S}, \forall I_j \in \mathcal{I} \\ z_{i,j} \in \{0, 1\}, \forall s_i \in \mathcal{S}, \forall I_j \in \mathcal{I} \\ z_{i,j}^l \in \{0, 1\}, \forall s_i \in \mathcal{S}, \forall I_j \in \mathcal{I}, \forall l \end{array} \right.$$

where $z_{j,i}^l$ stands for whether the l -th time slot of channel s_i is allocated to request I_j , and $O(v)$ denotes the objective function of the IP.

However, solving IP(IP (1)), IP(IP (2)), or IP(IP (3)) optimally is an NP-hard problem.

Theorem 1. *The optimal channel allocation problem is NP-hard in the Time-Window, Time-Window-Slice and Fixed-Interval cases.*

Proof. We consider a simple case that there is only one channel in the spectrum market and the time requests of secondary users are fixed intervals. This is a special case of Time-Window and Time-Window-Slice. In this case, our channel allocation problem is equivalent to the maximum weighted independent set problem, which is an NP-hard problem. This finishes the proof.

To tackle this NP-hardness, we will further design allocation mechanisms with performance guarantee to approximately optimize the social efficiency in Section 4.

4 SUBOPTIMAL SPECTRUM ALLOCATION: PVG

In this section, we propose a general spectrum allocation mechanism, *i.e.*, the Per-Value Greedy (PVG) spectrum allocation mechanism, to allocate the requests to channels efficiently for the Time-Window-Slice, the Time-Window and the Fixed-Interval cases.

4.1 The Framework of PVG

Here, we outline the framework of the PVG spectrum allocation mechanism. Recall that v_j is the weight (bid) of the request I_j , and t_j is the time length of I_j . The per-unit weight (bid) of request I_j can be calculated through $\eta_j = \frac{v_j}{t_j}$. All feasible requests in \mathcal{I} are sorted in the descending order of η_j . Algorithm 1 maintains a set \mathcal{A} of currently accepted requests. There are three possible cases in which request I_j can be accepted by Algorithm 1:

Case 1: When request I_j is considered according to the sorted order, we scan all the available channels one by one. If I_j can be allocated in one of these channels without overlapping with any other request in \mathcal{A} , $\text{Allocation_Saturated}(I_j, s_i) = \text{true}$, and I_j is immediately accepted.

Case 2: If I_j is not accepted in Case 1, we scan all the channels again to check if I_j can be allocated in one of them by deleting its overlapping requests. Suppose J_1, \dots, J_l are the requests which have been allocated in channel s_i

Algorithm 1 General Spectrum Allocation Framework**Input:**

$\mathcal{I} = \{I_1, \dots, I_N\}$ // \mathcal{I} : the set of all the requests in M_k sorted in descending order according to η_j ;

$\mathcal{S} = \{s_1, \dots, s_M\}$ // \mathcal{S} : the set of available spectrum in M_k ;

Output:

The set of accepted requests in \mathcal{A} ;

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1:  $\mathcal{A} = \emptyset$ ;
2: for  $j = 1$  to  $N$  do
3:   if  $v_j \geq \eta_s t_j$  then
4:     for  $i = 1$  to  $M$  do
5:       if  $Allocation\_Saturated(I_j, S_i) = true$  then
6:          $\mathcal{A} := \mathcal{A} \cup \{I_j\}$ ;
7:         accept  $I_j$  and  $Allocate(I_j, s_i)$ ;
8:         Break
9:       if  $I_j \notin \mathcal{A}$  then
10:        for  $i = 1$  to  $M$  do
11:          if  $Preemption\_Saturated(I_j, S_i) = true$  then
12:             $\mathcal{A} := \mathcal{A} \cup \{I_j\} \setminus \{J_1, \dots, J_n\}$ ;
13:            preempt  $\{J_1, \dots, J_n\}$  and  $Allocate(I_j, s_i)$ ;
14:            for  $k = 1$  to  $j$  do
15:              if  $I_k \notin \mathcal{A}$ ,  $v_k \geq \eta_s t_k$  and
                 $Allocation\_Saturated(I_k, S_i) = true$  then
16:                 $\mathcal{A} := \mathcal{A} \cup \{I_k\}$ ;
17:                accept  $I_k$  and  $Allocate(I_k, s_i)$ ;
18:                Break
19:            if  $I_j \notin \mathcal{A}$  then
20:              reject  $I_j$ ;
21: return  $\mathcal{A}$ ;
```

and overlap with I_j . If I_j 's weight is larger than β ($\beta \geq 1$) times of the total weight of J_1, \dots, J_l , $Preemption_Saturated(I_j, s_i) = true$, and I_j can also be accepted. Then, we add I_j in \mathcal{A} and delete J_1, \dots, J_h , and say that I_j “preempts” J_1, \dots, J_h .

On the other hand, if I_j 's weight is no larger than β times of the total weight of its overlapping requests J_1, \dots, J_l , $Preemption_Saturated(I_j, S_i) = false$, and we say that requests J_1, \dots, J_h directly “reject” I_j .

Case 3: Reaccept the rejected or preempted requests if there is no overlapping. After some other requests accepted in Case 2, request I_j which has

been rejected or preempted before can be reconsidered for acceptance if $Allocation_Saturated(I_j, s_i) = true$.

If a request is not accepted in Algorithm 1, it should be rejected or preempted by some other requests. In this paper, we say a request I_j "causes" the rejection or preemption of another request J , if either request I_j directly rejects or preempts request J , or preempts J indirectly. For example, if request J is preempted by request I , we say that I directly preempts J . After that, if request I is also preempted by I_j , we say that I_j preempts J indirectly.

Algorithm 1 is designed as a general spectrum allocation framework for the Time-Window-Slice, Time-Window and Fixed-Interval cases, however, the details of the preemption and allocation procedure are different in each of them. To this end, we will turn to the discussion of specific implementation details and performances for each cases in the following.

4.2 PVG for the Time-Window-Slice Case

Assume set $\{J_1, \dots, J_h\}$ includes all the requests which are allocated in the time slots from a_j to d_j of s_i . In order to check whether $Preemption_Saturated(I_j, S_i) = true$ in Algorithm 1, we first need to find out the requests that are allocated to channel s_i and overlapped with I_j . In the Time-Window-Slice case, we can find these requests as follows:

1. Compute the total time length T of the idle time slots in channel s_i . If $T < t_j$, we sort the set of requests $\{J_1, \dots, J_h\}$ according to their per-unit bids $\eta_k (1 \leq k \leq h)$.
2. Delete the request J_k with the smallest η_k from set $\{J_1, \dots, J_h\}$ and add it to the overlapping set of I_j . Suppose the time length of all the time slots that are allocated to J_k between a_j and d_j is t'_k . Then, we let $T = T + t'_k$.
3. Repeat step (2) until $T > t_j$.

If I_j 's weight is larger than β times of the total weight of the requests in I_j 's overlapping set, we delete all the overlapping requests from \mathcal{A} and allocate I_j in channel s_i .

If I_j is accepted in channel s_i , we allocate the time slots of s_i to request I_j by starting from I_j 's arrival time and searching for a series of available time slots in a backward manner. Theorem 2 gives the conclusion of approximation factor of the PVG allocation mechanism for the Time-Window-Slice case.

Theorem 2. *The approximation factor of the PVG is $6 + 4\sqrt{2}$ in the Time-Window-Slice case.*

Proof. Let \mathcal{O} be the set of requests chosen by the optimal mechanism \mathcal{OPT} , and \mathcal{A} be the set of requests accepted by Algorithm 1. For each request $I \in \mathcal{A}$, we define a set $R(I)$ of all the requests in \mathcal{O} that should be ‘‘accounted for’’ by I . $R(I)$ consists of I if $I \in \mathcal{O}$, and all the requests in \mathcal{O} which are rejected or preempted by I . More formally:

1. Assume I is accepted by case 1 or 3, then $R(I) = \{I\}$ in the case of $I \in \mathcal{O}$, and $R(I) = \emptyset$ otherwise.
2. Assume I is accepted by case 2, then $R(I)$ is initialized to contain all those requests from \mathcal{O} that were preempted (directly or indirectly) by I . In addition, $R(I)$ contains I in the case of $I \in \mathcal{O}$.
3. Assume $J \in \mathcal{O}$ is rejected by some other requests in Algorithm 1, and I_1, \dots, I_h are the requests in \mathcal{A} that should be accounted for the rejection of request J . Obviously, I_1, \dots, I_h and J are allocated in the same spectrum. Let v denote the weight of J and let v_j denote the weight of I_j for $1 \leq j \leq h$. We view J as h imaginary requests J_1, \dots, J_h , where the weight of J_i is $\frac{v_j v}{\sum_{j=1}^h v_j}$ for $1 \leq i \leq h$. $R(I_j) := R(I_j) \cup \{J_j\}$. Note that the weight of J_j is no larger than β times the weight of I_j according to the rejection rule.

For each request $J \in \mathcal{O}$, if $J \in \mathcal{A}$, it has been included in $R(J)$. Otherwise, it must be preempted or rejected by some request $I \in \mathcal{A}$, and then J belongs exactly to the set $R(I)$. Thus, the union of all these sets for $I \in \mathcal{A}$ covers \mathcal{O} .

We now fix a request $I \in \mathcal{A}$. Let v be the weight of I and let V be the total weight of all requests in $R(I)$. Then, we can get that $V = v' + v'' + v$ if $I \in \mathcal{O}$; otherwise, $V = v' + v''$. Here, v' denotes the total weight of all requests preempted by I , and v'' is the total weight of all (or partial) requests rejected by I . Therefore, we can conclude that $V \leq v' + v'' + v$. Define $\rho = V/v$. Our goal is to give the upper bound of ρ .

We first consider the requests that have been rejected by I . According to line 20 of Algorithm 1, if $J \in \mathcal{O}$ overlaps with requests I_1, \dots, I_h , we split J into h imaginary requests J_1, \dots, J_h , and let each overlapping request I_j account for an imaginary request J_j . Therefore, we can assume that each J_j is only rejected by I_j . On the other hand, if we remove I from \mathcal{A} , all of the requests or imaginary requests rejected by I can be accepted by \mathcal{A} .

Let J_1, \dots, J_q be the requests in \mathcal{O} which were rejected by I . If there exists a request J_k ($1 \leq k \leq q$) which can partition J_1, \dots, J_q into two disjoint sections in time-axis, we define the arrival point of J_k a critical point; otherwise, we choose the arrival time of request I as the critical point. The whole time-axis be classified into *LOS* (Left of Separatrix) and *ROS* (Right of

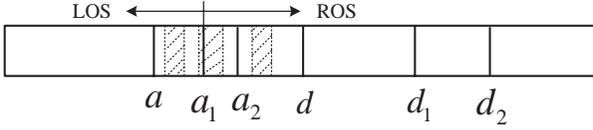


FIGURE 2

The whole time-axis is classified into *LOS* (Left of Separatrix) and *ROS* (Right of Separatrix) by using the critical point as the separatrix. Arrival time a_1 of request J_1 is the critical point in this instance. Shadow parts indicate the time slots occupied by I .

Separatrix) by using the critical point as the separatrix. We define the request whose arrival time is later than critical point belongs to *ROS*; otherwise, the request belongs to *LOS*.

Assume that the time length of I is t , the allocated times in *LOS* and *ROS* last t' and t'' , respectively. We can easily get that:

$$t = t' + t''$$

Assume that there exists two requests J_1 and J_2 which are located at *ROS* in Figure 2, and a_1 is the critical point. Since we allocate time slots for a request starting from its arrival time and search for continuous available time slots in a backward manner, J_2 being rejected by I indicates that all the available time from a_2 to d_2 is less than the time length of J_2 when $d_1 \leq d_2$. However, J_1 and J_2 can be accepted while removing I from \mathcal{A} . Hence, the time length of J_1 is at most equal to the time from a_1 to d . Assume that the overlapping time between request J_k and request I is t'_k . If all the requests that have the overlapping time with J_1 account for the corresponding weight, we can cover the total weight of J_1 . Therefore, the weight of I that should account for J_1 is equal to $\eta_1 t'_1$. It's obvious that $t'_1 < t''$. Thus, we can calculate V_{ROS} which is defined as the total weight of all the requests I that should account for in *ROS*:

$$V_{ROS} \leq v_2 + \eta_1 t'_1 \leq v_2 + \eta_1 t''$$

When $d_1 > d_2$, we demonstrate that $V_{ROS} \leq v_1 + \eta_2 t'_2 \leq v_1 + \eta_2 t''$ similarly.

Assuming that there are more than two requests in \mathcal{A} which are located in *ROS* and were rejected by I , we can easily conclude that $V_{ROS} \leq v_i + \sum_{k \neq i} \eta_k t'_k$, and $\sum_{k \neq i} t'_k \leq t''$.

According to the rejection rule, we can get:

$$v_i \leq \beta v$$

Since the η_k from any rejected request J_k is less than the value of η from I , the following holds:

$$\sum_{k \neq i} \eta_k t'_k \leq \sum_{k \neq i} \eta t'_k \leq \eta t''$$

Thus, the total weight V_{ROS} of all the rejected requests in ROS satisfies:

$$V_{ROS} \leq \beta v + t'' \eta$$

By the same method, we can easily find the total weight V_{LOS} of all the rejected requests in LOS satisfies:

$$V_{LOS} \leq \beta v + t' \eta$$

Combining the above two conditions, the total weight of all the requests can be calculated:

$$v'' = V_{ROS} + V_{LOS} \leq 2\beta v + (t' + t'')\eta = 2\beta v + v$$

We now assume inductively that the ρ bound is valid for time with a larger per-unit weight than that of I . Since the overall weight of the time that directly preempted by I is at most v/β , we can get $v/\beta * \rho \geq v'$. Recall that $V \leq v' + v'' + v$ and $v'' \leq 2\beta v + v$ hold. We can obtain that $V \leq v\rho/\beta + 2\beta v + v + v$. This implies that $V/v = \rho \leq \rho/\beta + 2\beta + 2$. The inequality can be depicted as $\rho \leq \frac{2(\beta+1)}{1-1/\beta}$ equivalently. ρ takes its minimal value when $\beta = 1 + \sqrt{2}$, which implies that $\rho \leq 6 + 4\sqrt{2}$. Finally, since the ρ bound holds for all the requests in \mathcal{A} and the union of all $R(I)$ sets covers all the requests taken by OPT , we can conclude that the $EFF(OPT)$ is at most ρ times of the social efficiency $EFF(\mathcal{A})$. Therefore, the approximation factor is $6 + 4\sqrt{2}$.

4.3 PVG for the Time-Window Case

In the Time-Window case, we search the overlapping set of request I_j in channel s_i as follows:

1. We first find the longest idle time slot within a_j and d_j , and let T be the length of this time slot.
2. Assume that the set $\{J_1, \dots, J_h\}$ includes all the requests which are allocated to s_i in the time slots between a_j and d_j . Then, we search the request $J_k \in \{J_1, \dots, J_h\}$, which has the smaller per-unit bid than the

other and is allocated in the time slot adjacent to the longest idle time slot. We add J_k to I_j 's overlapping set, and extend the length of the longest idle time slot by regarding the time slots, that are allocated to the requests in I_j 's overlapping set, as being idle.

3. Repeat step (2) until the extended length of the longest idle time slot is longer than the requested time length of I_j .

If $\text{Preemption_Saturated}(I_j, S_i) = \text{true}$ in the Time-Window case, we delete all I_j 's overlapping request in channel s_i , and we allocate I_j in the first idle time slot between a_j and d_j of channel s_i , which is longer than the time length I_j requested.

Theorem 3. *The approximation factor of the PVG is 8 in the Time-Window case.*

Proof. Similar to the analysis of theorem 2, We construct the set $R(I)$. The conclusion on that the union of all sets $R(I)$ for $I \in \mathcal{A}$ covers \mathcal{O} still holds. Thus, we still focus on a single request $I \in \mathcal{A}$ to find the upper bound of ρ .

Recall that v is the weight of I , and V is the total weight of all requests in $R(I)$. Then, we can get that $V = v + V'$ if $I \in \mathcal{O}$, where V' denotes the total weight of the requests in $R(I)$ but not in \mathcal{A} ; otherwise, $V = V'$. Therefore, we can conclude that $V \leq v + V'$.

Assuming that request $J \in \mathcal{O}$ but $J \notin \mathcal{A}$, and request $I \in \mathcal{A}$ should be accounted for the rejection or preemption of request J in the PVG allocation mechanism. Suppose that the time slots allocated to the requests J and I are (st_J, et_J) and (st_I, et_I) , respectively. Then, the relationships between the two time slots can be grouped into three types as shown in Figure 3. We now discuss the three complementary types respectively.

Type 1: $st_J < st_I < et_J$, e.g. J_1

Obviously, all the requests belonging to Type 1 are overlapping with each other in time domain. Thus, there is at most one request which belongs to this type in the Time-Window case. Let V_1 be the weight of the request belonging to Type 1. We get that $V_1 \leq \beta v$ according to the preemption rule.

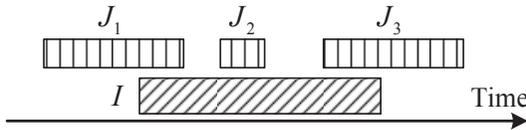


FIGURE 3

There exist three types of requests that overlap with request I . Jobs J_1 , J_2 and J_3 are used to illustrate type 1, type 2 and type 3, respectively.

Type 2: $st_J \geq st_I$ and $et_J \leq et_I$, e.g. J_2

For Type 2, it may contain several requests, denoted by J_1, J_2, \dots, J_k , and these requests can be further divided into three subtypes:

The first subtype is $\eta_{J_i} \leq \eta_I$. In this case, for each time slot allocated to request I , there is at most one request sharing it with request I . Thus, the total weight of requests in this subtype, denoted by V'_2 , is at most v , i.e. $V'_2 \leq v$.

The second subtype is $\eta_{J_i} \geq \eta_I$ and J_i is preempted by request I (directly or indirectly). According our preemption rule, the total weight of these requests (denoted by V''_2) is at most v/β , i.e. $V''_2 \leq v/\beta$.

The third subtype is $\eta_{J_i} \geq \eta_I$ and J_i is rejected by request I indirectly, that is, request J_i is rejected by a request that is preempted by request I . The total weight of this subtype requests, denoted by V'''_2 , is at most $(2\beta + 1)$ times of the total weight of the requests preempted by I , which is at most v/β . Thus, we get that $V'''_2 \leq (2\beta + 1)v/\beta$.

Type 3: $et_I < et_J$, e.g. J_3

Let V_3 denote the total weight of the requests belonging to Type 3. The analysis of this type is similar with Type 1. Therefore, we can also easily get that $V_3 \leq \beta v$ holds.

From above, the sum weight of requests in $R(I)$ can be calculated by $V \leq v + V_1 + V_3 + V'_2 + V''_2 + V'''_2 = v + 2\beta v + v + v/\beta + (2\beta + 1)v/\beta = 2v(2 + \beta + 1/\beta)$. Therefore, $\rho = V/v \leq 2(\beta + 1/\beta + 2) \leq 8$, where $\beta + 1/\beta$ is maximized when $\beta = 1$, i.e. the approximation factor is 8.

4.4 PVG for the Fixed-Interval Case

In the Fixed-Interval case, it is easy to judge which requests have been allocated in a channel are conflict with a fixed request I . Thus, its not a hard work to check whether $Preemption_Saturated(I_j, S_i) = true$ and allocate a request in a channel. In the following, we will directly give the analysis of approximation factor of the PVG allocation mechanism for the Fixed-Interval case.

Theorem 4. *The approximation factor of the PVG is 32 in the Fixed-Interval case.*

Proof. Similar to the analysis of theorem 3, We construct the set $R(I)$ for each $I \in \mathcal{A}$ to find the upper bound of ρ .

Recall that (a_I, d_I) and (a_J, d_J) are the arrival times and deadlines of requests I and J . For an arbitrary request $I \in \mathcal{A}$, the requests it should account for can be categorized into five types.

Type 1: Request I should account for the rejection of $J \in \mathcal{O}$, and $a_J < a_I$. Suppose there are N_1 requests $\{J_1, J_2, \dots, J_{N_1}\}$ that belong to type 1. Since all of these requests belong to \mathcal{O} and conflict with each other in time domain, they must conflict-free in spatial domain. We can take location of the request as the center of the interference circle. Then, for an arbitrary couple of requests J_1 and J_2 , we can get that $\angle L_{J_1} L_I L_{J_2} > \pi/3$. Therefore, we can conclude that $N_1 \leq 5$.

If J is rejected by more than one request in \mathcal{A} , we also let each of these requests only account for part of the bid of J . Recall that there are at most 5 requests belong to type 1 for request I . Then, we can easily conclude that the total bid of requests that belong to type 1 is less than 5 times of v .

Type 2: Request I should account for the rejection of $J \in \mathcal{O}$, and $d_J > d_I$. We can get the same conclusion through the analysis as that of type 1. Therefore, the sum of requests that belong to type 2 is less than 5 times of v .

Type 3: Request I should account for the rejection of $J \in \mathcal{O}$, and $a_J > a_I$, $d_J < d_I$. Since J is rejected by I , and $\eta_J \leq \eta_I$ according to *Algorithm 1*. There are at most 5 requests in the optimal solution share each time slice allocate to I . Then we can get that the total bids of requests that belong to type 3 is less than the 5 times of v .

Type 4: Request I should account for the preemption of $J \in \mathcal{O}$. It's obvious that the sum of requests that belong to type 4 is less than v according to our preemption rule.

Type 5: $J \in \mathcal{O}$ is rejected by the requests that are preempted by I . In this case, we say that J is rejected by I indirectly. The total bid of requests that belong to type 5 is less than 15 times (the total bid of type 1, 2 and 3) of the bids of requests that directly reject them. On the other hand, the total bid of requests directly reject them been preempted by I directly or indirectly, which is less than the bid of I . Thus, we can get that the total bid of requests that belong to type 5 is less than 15 times of the bid of I .

From the above analysis, ρ can be calculated by $\rho \leq \frac{(1+5+5+5+1+15)v}{v} = 32$. Since the ρ bound holds for all the requests in I , the approximation factor is 32.

Now, we straightforwardly present the time complexity of the PVG:

Theorem 5. *The time complexity of the PVG is $O(MN^3)$, where M is the number of channels, and N is the number of requests.*

5 SIMULATION RESULTS

In this section, we conduct extensive simulations to examine the performance of the proposed PVG allocation mechanism.

5.1 Simulation Setup

In order to make the experimental results more convincing and close to the reality, we adopt the data set obtained through the analysis of measurement data which are collected in Guangdong Province, China. We choose the frequency band of *Broadcasting TVI*(48.5 - 92MHz) for comparison from many frequency bands of services, and capture continuous 5 days' records from the whole measurement data. The total bandwidth of *TVI* is split into plenty of channels in accordance with the width of 0.2MHz. For each channel, the data are divided into massive time slots, and we roughly set each time slot about 75 seconds. As a result, the total number of time slots reaches to 5760 (5days/75s).

Figure 4 shows a depiction of the channel vacancies located in frequency band of *TVI*. We use black color to represent the occupied time slots and white color to denote the white space for each channel. The spectrum usage figure makes some characteristics of spectrum usage easier to visualize, for instance, we can easily find that the usage time of the primary user is basically the same in each day. Therefore, the vacancy time slots in all 5 days are selected as the idle slots for allocation to ensure the usage of primary user at the same time.

In our simulations, we select 3 channels from the whole frequency band of *TVI* as input, and the total time of each channel lasts 24 hours from 0:00 to 24:00. We generate secondary users' bid values randomly from the range of $[0, 1)$ and the time length of each secondary user uniformly from the range of $[0.5, 2]$ hours. The request time slot with arrival time and deadline for each secondary user is uniformly distributed in the range of $[2, 4]$ hours. λ shows the number of requests in our setting, Here, we generate two different scenarios.

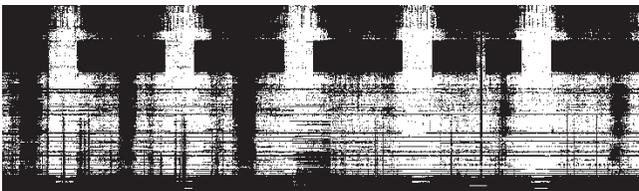


FIGURE 4
Usage of spectrum for 5 days, an instance of frequency band of *Broadcasting TVI*.

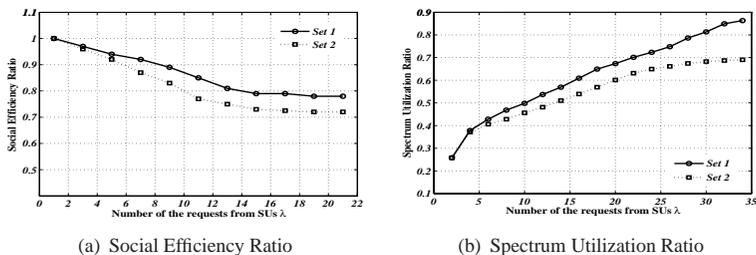


FIGURE 5

Social efficiency ratio and spectrum utilization ratio under Sets 1 and 2 in the Time-Window-Slice Case, $\eta_s = 0$, $\beta = 2$.

- **Set 1:** All the requests are uniformly distributed in 24 hours without hot time.
- **Set 2:** There exists hot time in this setting, which contains about δ requests of the whole day. In our simulation setting, we set $\delta = 80\%$.

5.2 Performance of the PVG allocation Mechanism

In this section, we study the performance of the PVG mechanism compared with the optimal allocation mechanisms in Time-Window-Slice, Time-Window and Fixed-Interval cases. For each case, we mainly focus on the performance of social efficiency and the utilization of channels. For comparison, we plot the results under 2 different request sets mentioned above, and analyze influences of the relationship between supply and demand from the results.

5.2.1 Performance of Time-Window-Slice and Time-window cases

In the following, Figures. 5 and 6 show the simulation results in the Time-Window-Slice and Time-Window cases respectively. Because we use the

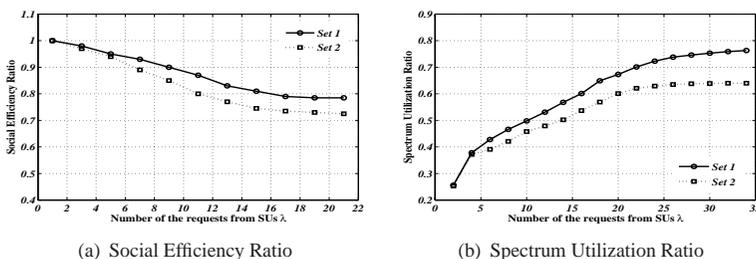


FIGURE 6

Social efficiency ratio and spectrum utilization ratio under Sets 1 and 2 in the Time-Window Case, $\eta_s = 0$, $\beta = 2$.

same setting in these two cases (both of them only allow requests share channels in temporal domain), we can see that the results in both cases are roughly similar. Thus, as a representative, our discussion below is mainly based on the results in the Time-Window-Slice case.

Now, let us look at the achieved social efficiency and spectrum utilization of the proposed mechanisms. Figure 5(a) illustrates the social efficiency ratio of the PVG allocation mechanism to the optimal allocation mechanism in the Time-Window-Slice case. We see that the PVG mechanism works as well as the optimal mechanism when λ is small. This is because there is enough available time for each request, and most of them can be allocated without overlapping with others in both schemes. Since the competition among requests increases as λ increases, the optimal allocation mechanism outperforms the PVG allocation mechanism gradually. The social efficiency ratio keeps approximately stable when λ is large enough, that is, the supply is much less than the demand. In Figure 5(b), the spectrum utilization ratio of the two allocation mechanisms in the Time-Window-Slice case is also depicted. It is shown that the utilization ratio simply increases along with the increasing of λ . From Figure 5(a), we can see that the PVG allocation mechanism performs better in a lightly loaded system than in a highly loaded system. Furthermore, even in the worst case, the social efficiency ratio of the PVG allocation mechanism to the optimal allocation mechanism is still above 70%. In the Time-Window case, Figure 6 shows exactly the same trends in spectrum utilization ratio.

5.2.2 Performance of Fixed-Interval case

Figure 7(a) shows the social efficiency ratio of the PVG allocation mechanism in the Fixed-Interval case. Unsurprisingly, the performances of PVG in the Time-Window-slice and Time-Window cases are better than that of

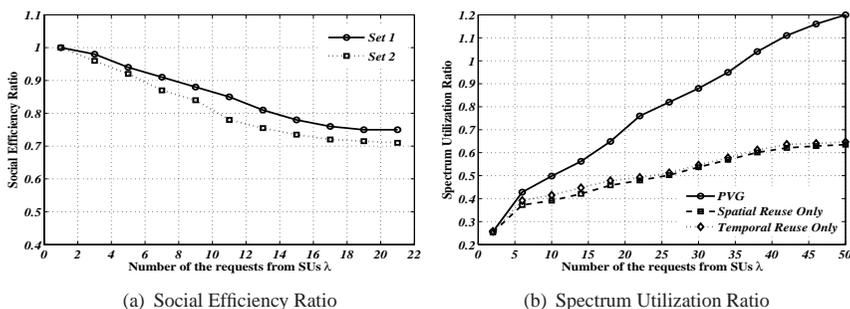


FIGURE 7

Social efficiency ratio under Sets 1 and 2 in the Fixed-interval Case and the comparison of spectrum utilization ratio under different reuse ways in Sets 1, $\eta_s = 0$, $\beta = 2$.

PVG in the Fixed-Interval case. This is because we always get a solution whose value is larger than $1/(6 + 4\sqrt{2})$ and $1/8$ times of the optimal one in the Time-Window-slice and Time-Window cases, while the solution is only always larger than $1/32$ times of the optimal one in the Fixed-interval case. However, our simulation results are much better than the theoretical bound we proved in previous sections.

Different with other two cases, channels can be reused both in spatial and temporal domain in the Fixed-Interval case. In Figure 7(b), we compare the spectrum utilization efficiency in the case of Fixed-Interval with the case of channels reusable only in spatial, or temporal domain. We define spectrum utilization ratio to be the ratio of the time length allocated to winners and the total time length of all the channels available. Obviously, the spectrum utilization efficiency in our model is much better than the cases that spectrum can only be reused in spatial or temporal domain.

6 LITERATURE REVIEWS

With the fast growing spectrum-based services and devices in recent years, the remaining spectrum available for future wireless services and devices is being exhausted. To tackle this problem, it has been extensively studied in the scope of the secondary spectrum allocation problem. Many designs of wireless spectrum allocation mechanisms have been proposed to cope with dynamic spectrum access problem in various scenarios. For instance, Huang *et al.* [15] comparatively early discussed secondary spectrum allocation mechanisms for sharing spectrum among a group of users. In order to maximize the total utility and minimize the interference [26], transmit power based channel allocation methods were proposed. At the same time, [9] and [22] studied the spectrum band allocation methods aiming to minimize the spectrum interference.

Spectrum can be reused both in spatial and temporal domain. Most of the existing studies of spectrum allocation only allow the secondary users share one channel in spatial domain, such as [4, 10, 11, 13, 16, 23, 25, 27, 28, 34, 35]. In spectrum allocation mechanism design, another line is based on spectrum temporal reuse (*e.g.* [7, 24, 29, 31]). TODA [24] first takes time domain into account, and proposes a suboptimal spectrum allocation mechanism with polynomial time complexity aiming to generate maximum revenue for the auctioneer. There are only few studies allow spectrum reused both in spatial and temporal domain, *e.g.* [12, 14]. In addition, spectrum is a local resource. Thus, District mechanism [25] takes the spectrum locality into account and gives an economically robust and computationally efficient method. Unfortunately, none of the existing studies consider the Time-Window or

Time-Window-Slice cases in the spectrum allocation mechanism design, which are the more general cases than the Fixed-interval case.

Different from traditional periodic model (offline model), many researches study the spectrum allocation in an online model [5, 24, 30–32, 36]. As for online spectrum allocation, the requests from secondary users can be arrived at anytime [33]. Deek *et al.* [5] studied a time-based cheating problem in an online model. However, a significant issue is not fully addressed in the mentioned previous designs, most existing works concentrate on spectrum allocation mechanism design without considering spectrums as non-identical items.

These years, some work on heterogeneous spectrum transaction issue has been studied in [7, 8, 18, 20, 21]. In [8], Feng *et al.* propose a method for heterogeneous spectrum allocation. [18] and [21] solve the heterogeneous auction problems in different perspectives. Nevertheless, they do not consider time domain issue in their works, thus making the spectrum allocation incomplete. Similar work is proposed in [7], Dong *et al.* tackled the spectrum allocation problem by using combinatorial auction model with consideration of time-frequency flexibility. The spectrum opportunity is divided into many fractions in time and frequency domains. However, the fixed candidate slots division model will greatly restrict the choice flexibility for secondary users.

7 CONCLUSION

Considering the challenges for designing a practical spectrum allocation in wireless ad hoc networks, we proposed a series of spectrum allocation mechanisms to maximize the social efficiency for the cases of Time-window-slice, Time-window and Fixed-interval. We first studied the optimal spectrum allocation problem in secondary spectrum market, which is an NP-hard problem. Then, we designed a general framework of spectrum allocation, namely PVG. The proposed PVG scheme has a constant approximation factor but is computationally much more efficient. To the best of our knowledge, this is the first work that takes a flexible time request from secondary users into consideration.

Several interesting questions are left for future research. The first one is to study the case when the request of a secondary user may be served by several channels in the spectrum market. The second challenging question is to design truthful mechanisms when the requests arrive online and we have to make online decisions on allocations and payment.

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