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Interval Type-2 Fuzzy Analytic Network Process for Modelling a Third-party Logistics (3PL) Company

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The Fuzzy Analytic Network Process (ANP) is generally used for solving multi- criteria decision making (MCDM) problems by considering the pairwise comparison between criteria/sub-criteria, and inner/outer dependencies among criteria. Linguistic expressions are used for experts' judgements, and these judgements are imprecise and vague. Hence, incorporating fuzziness with multi-criteria decision making techniques is as advanced approach as fuzzy AHP/ANP. Additionally, type-2 fuzzy sets are modelled with vagueness considering the fuzziness of a membership function. Although fuzzy AHP/ANP methods are widely used for MCDM problems, few studies are available in the literature with type- 2 fuzzy AHP. Therefore, the type- 2 fuzzy ANP method is first introduced in this paper with interval type-2 fuzzy sets.

The main goal of this paper was to develop a new approach for the interval type-2 fuzzy ANP method for modelling MCDM problems by integrating ANP and interval type-2 fuzzy sets. 3PL company selection problems were modelled with an interval type-2 fuzzy ANP method with BOCR main criteria.

Keywords: Type-2 fuzzy sets, fuzzy analytic network process, type-2 FANP.

1 INTRODUCTION

Many methods have been presented for handling fuzzy multiple attribute decision problems. Laarhoven and Pedrycz [1] proposed a fuzzy logarithmic

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least squares method to obtain fuzzy weights from triangular fuzzy comparison matrices. Chang [2] suggested an extent analysis method. Buckley [3] offered the geometric mean method to calculate fuzzy weights. Wang and Hwang [4] introduced linear, non-linear, dynamic, goal, and stochastic mathematical programming for the Research and Development project selection. Chen and Lee [5] used interval type-2 fuzzy sets to define linguistic variables and proposed likelihood approaches.

The ANP method, which was developed by Saaty [6, 7], is one of the multi-criteria decision making methods for complex models. Because there are qualitative criteria, and interactions among the criteria as well as the linguistic variables, fuzzy ANP which is a combination of ANP and Fuzzy Logic methods was developed. Therefore, in the literature, fuzzy ANP methods are based on type-1 fuzzy sets. Fuzzy ANP has a many advantages over classical ANP. Thus, fuzzy ANP is a more effective method for eliminating uncertainty in opinions of decision makers' and judgements from classical ANP.

In type-1 fuzzy sets, uncertainties are handled with varying degrees of membership between 0 and 1. If the value is assigned a value of 0, the element does not belong to the fuzzy set. If the value is assigned as 1 the element does belong to the fuzzy set. If the value is assigned as 0.5, the element belongs 50 percent to fuzzy set. However, words mean different things to different people, so there is uncertainty related to words, which means that fuzzy logic must somehow use this uncertainty when it computes with words. Type-1 fuzzy logic cannot do this, but type-2 fuzzy logic, as recently defended by Karnik and Mendel [8, 9], and Mendel [10]. The concept of a type-2 fuzzy set was presented by Zadeh [11] as an extension of the concept of an ordinary fuzzy set called a type-1 fuzzy set. Type-1 fuzzy sets are two dimensional, but type-2 fuzzy sets are there dimensional. When we cannot determine the membership grade even as an exact number in [0,1] , we use fuzzy sets of type-2. Dubois and Prade [12], Karnik and Mendel [8, 9], Kaufman and Gupta [13], Mizumoto and Tanaka [14, 15], Turksen [16], and Yager [17] have contributed to the literature to develop type-2 fuzzy sets.

Interval type-2 fuzzy sets are a special condition of generalized type-2 fuzzy sets. Because of the computational complexity of using general type-2 fuzzy sets, most people use interval type-2 fuzzy sets as a type-2 fuzzy sets, the result being an interval fuzzy sets. Therefore, we used interval type-2 fuzzy sets [18].

In this paper, an interval type-2 fuzzy ANP method was developed and offered into the literature for the first time. Experts compare criteria according to the linguistic scale fuzzy ANP as type-2 fuzzy trapezoidal numbers.

The rest of these paper is organized as follows. Section 2 presents the basics of interval fuzzy sets. Arithmetic operations with trapezoidal interval type-2 fuzzy sets are given in Section 3. Section 4 presents type reduction for type-2 fuzzy sets. Section 5 presents our proposed interval type-2 fuzzy ANP method. Section 6 provides an application for solving the supplier selection problem in Turkey. Finally Section 7 gives the conclusions.

2 INTERVAL TYPE-2 FUZZY SETS

In this section, interval type-2 fuzzy sets are first explained [19]. Second, defuzzification methods are presented.

Definition 2.1. *A type-2 fuzzy set $\tilde{\tilde{A}}$ is characterized by a type-2 membership function, a type-2 membership function $\mu_{\tilde{\tilde{A}}}$, shown as follows [5]:*

$$\tilde{\tilde{A}} = \left\{(x,u), \mu_{\tilde{\tilde{A}}}(x,u) | \forall x \in X, 0 \le \mu_{\tilde{\tilde{A}}}(x,u) \le 1\right\} \tag{1}$$

where $\forall u \in J_X \subseteq [0,1]$*. The type-2 fuzzy set $\tilde{\tilde{A}}$ is expressed as follows:*

$$\tilde{\tilde{A}} = \int_{x\in X} \int_{u \in J_x} \mu_{\tilde{\tilde{A}}}(x,u)/(x,u) \tag{2}$$

where x is the primary variable in the domain X; u is the secondary variable in domain J_X at each $x \in X$. J_X is called the primary membership of x, and the secondary membership grades of $\tilde{\tilde{A}}$ all equal to 1, $J_X \subseteq [0,1]$ and $\iint$ denote union over all admissible x and u. For discrete universes of discourse, $\int$ is replaced by $\sum$.

An interval type-2 fuzzy set $\tilde{\tilde{A}}$ is a special case of general type-2 fuzzy sets where all of the secondary membership functions of $\tilde{A}$ are equal to 1. $\tilde{\tilde{A}}$ is an interval type-2 fuzzy sets

$$\tilde{\tilde{A}} = \int_{x\in X} \int_{u \in J_x} 1/(x,u) \tag{3}$$

where $J_X \subseteq [0,1]$.

Uncertainty in the primary memberships of $\tilde{\tilde{A}}$ is defined as a footprint of uncertainty (FOU). FOU defines the union of all primary memberships as [19]:

$$FOU(\tilde{\tilde{A}}) = \bigcup_{x\in X} J_X = \{(x,u) : u \in J_X \subseteq [0,1]\} \tag{4}$$

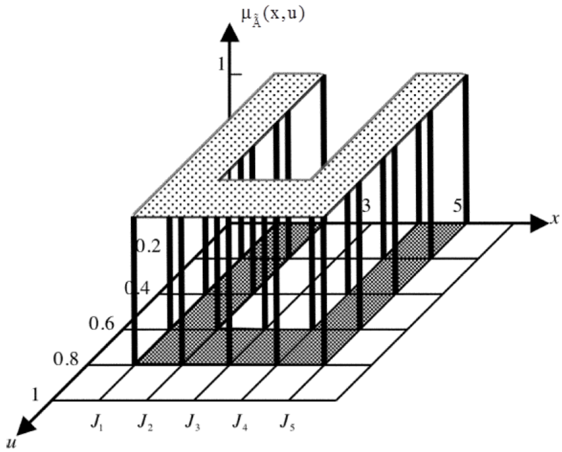

FIGURE 1
Example of an interval type-2 membership function for discrete universes of discourse [19]

The lower and upper bounds of FOU of $\tilde{\tilde{A}}$ are two type-1 membership functions named the Lower Membership Function (LMF), $\underline{\mu}_{\tilde{\tilde{A}}}(x)$, and the Upper Membership Function (UMF), $\bar{\mu}_{\tilde{\tilde{A}}}(x)$, respectively.

$$\bar{\mu}_{\tilde{\tilde{A}}}(x) = \overline{FOU(\tilde{\tilde{A}})}, \forall x \in X \tag{5}$$

$$\underline{\mu}_{\tilde{\tilde{A}}}(x) = \underline{FOU(\tilde{\tilde{A}})}, \forall x \in X \tag{6}$$

$$FOU(\tilde{\tilde{A}}) = \bigcup_{x \in X} \left[\underline{\mu}_{\tilde{\tilde{A}}}(x), \bar{\mu}_{\tilde{\tilde{A}}}(x)\right] \tag{7}$$

3 ARITHMETIC OPERATIONS BETWEEN TRAPEZOIDAL INTERVAL TYPE-2 FUZZY SETS

Arithmetic operations with trapezoidal interval type-2 fuzzy sets are given as follows.

Definition 3.1. *The upper membership function and lower membership function of an interval type-2 fuzzy set are type-1 membership functions, respectively, a trapezoidal interval type-2 fuzzy set*

$$\tilde{A}_i = \left(\tilde{A}_i^U, \tilde{A}_i^L\right) = \begin{pmatrix} \left(a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U; H_1\left(\tilde{A}_i^U\right), H_2\left(\tilde{A}_i^U\right)\right) \\ \left(a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L; H_1\left(\tilde{A}_i^L\right), H_2\left(\tilde{A}_i^L\right)\right) \end{pmatrix}$$

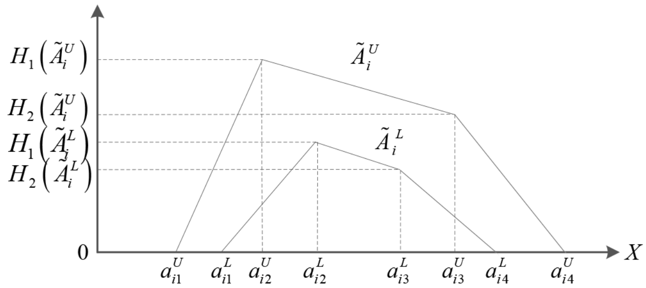

FIGURE 2
The membership functions of the interval type-2 fuzzy set $\tilde{\tilde{A}}$

where $\tilde{A}_i^U$ *and* $\tilde{A}_i^L$ *are type 1 fuzzy sets,* $a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U, a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L$ *are the references points of the interval type-2 fuzzy* $\tilde{\tilde{A}}; H_j(\tilde{A}_i^U)$ *denotes the membership value of the element* $a_{i(j+1)}^U$ *in the upper trapezoidal membership function* $\tilde{A}_i^U$*;* $1 \leq j \leq 2$, $H_j(\tilde{A}_i^L)$ *denotes the membership value of the element* $a_{i(j+1)}^U$ *in the upper trapezoidal membership function* $\tilde{A}_i^U$*; and* $1 \leq j \leq 2$, $H_j(\tilde{A}_i^U)$, $H_j(\tilde{A}_i^L) \in [0, 1]$, $1 \leq i \leq n$

Definition 3.2. *The addition operation between two trapezoidal interval type-2 fuzzy sets*

$$\tilde{A}_1 = (\tilde{A}_1^U, \tilde{A}_1^L) = \begin{pmatrix} \left(a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1\left(\tilde{A}_1^U\right), H_2\left(\tilde{A}_1^U\right)\right) \\ \left(a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; H_1\left(\tilde{A}_1^L\right), H_2\left(\tilde{A}_1^L\right)\right) \end{pmatrix}$$

$$\tilde{A}_2 = (\tilde{A}_2^U, A_2^L) = \begin{pmatrix} \left(a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; H_1\left(\tilde{A}_2^U\right), H_2\left(\tilde{A}_2^U\right)\right) \\ \left(a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; H_1\left(\tilde{A}_2^L\right), H_2\left(\tilde{A}_2^L\right)\right) \end{pmatrix}$$

is defined as follows [5, 20]:

$$\tilde{\tilde{A}}_1 \oplus \tilde{\tilde{A}}_2 = \left(\tilde{A}_1^U, \tilde{A}_1^L\right) \oplus \left(\tilde{A}_2^U, \tilde{A}_2^L\right) = \left(\begin{array}{l} \begin{pmatrix} a_{11}^U + a_{21}^U, a_{12}^U + a_{22}^U, a_{13}^U + a_{23}^U, a_{14}^U + a_{24}^U; \\ \min\left(H_1\left(\tilde{A}_1^U\right), H_1\left(\tilde{A}_2^U\right)\right) \min\left(H_2\left(\tilde{A}_1^U\right), H_2\left(\tilde{A}_2^U\right)\right) \end{pmatrix}, \\ \begin{pmatrix} a_{11}^L + a_{21}^L, a_{12}^L + a_{22}^L, a_{13}^L + a_{23}^L, a_{14}^L + a_{24}^L; \\ \min\left(H_1\left(\tilde{A}_1^L\right), H_1\left(\tilde{A}_2^L\right)\right), \min\left(H_2\left(\tilde{A}_1^L\right), H_2\left(\tilde{A}_2^L\right)\right) \end{pmatrix} \end{array} \right) \tag{8}$$

Definition 3.3. *The subtraction operation between the trapezoidal interval type-2 fuzzy sets*

$$\tilde{\tilde{A}}_1 = \left(\tilde{A}_1^U, \tilde{A}_1^L\right) = \begin{pmatrix} \left(a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1\left(\tilde{A}_1^U\right), H_2\left(\tilde{A}_1^U\right)\right), \\ \left(a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; H_1\left(\tilde{A}_1^L\right), H_2\left(\tilde{A}_1^L\right)\right) \end{pmatrix}$$

$$\tilde{\tilde{A}}_2 = \left(\tilde{A}_2^U, \tilde{A}_2^L\right) = \begin{pmatrix} \left(a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; H_1\left(\tilde{A}_2^U\right), H_2\left(\tilde{A}_2^U\right)\right), \\ \left(a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; H_1\left(\tilde{A}_2^L\right), H_2\left(\tilde{A}_2^L\right)\right) \end{pmatrix}$$

is defined as follows [5, 20]:

$$\tilde{\tilde{A}}_1 \ominus \tilde{\tilde{A}}_2 = \left(\tilde{A}_1^U, \tilde{A}_1^L\right) \ominus \left(\tilde{A}_2^U, \tilde{A}_2^L\right) = \\ \left(\begin{array}{c} \begin{pmatrix} a_{11}^U - a_{24}^U, a_{12}^U - a_{23}^U, a_{13}^U - a_{22}^U, a_{14}^U - a_{21}^U; \\ \min\left(H_1\left(\tilde{A}_1^U\right), H_1\left(\tilde{A}_2^U\right)\right), \min\left(H_2\left(\tilde{A}_1^U\right), H_2\left(\tilde{A}_2^U\right)\right) \end{pmatrix}, \\ \begin{pmatrix} a_{11}^L - a_{24}^L, a_{12}^L - a_{23}^L, a_{13}^L - a_{22}^L, a_{14}^L - a_{21}^L; \\ \min\left(H_1\left(\tilde{A}_1^L\right), H_1\left(\tilde{A}_2^L\right)\right), \min\left(H_2\left(\tilde{A}_1^L\right), H_2\left(\tilde{A}_2^L\right)\right) \end{pmatrix} \end{array} \right) \tag{9}$$

Definition 3.4. *The multiplication operation between the trapezoidal interval type-2 fuzzy sets*

$$\tilde{\tilde{A}}_1 = \left(\tilde{A}_1^U, \tilde{A}_1^L\right) = \begin{pmatrix} \left(a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1\left(\tilde{A}_1^U\right), H_2\left(\tilde{A}_1^U\right)\right), \\ \left(a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; H_1\left(\tilde{A}_1^L\right), H_2\left(\tilde{A}_1^L\right)\right) \end{pmatrix}$$

$$\tilde{\tilde{A}}_2 = \left(\tilde{A}_2^U, \tilde{A}_2^L\right) = \begin{pmatrix} \left(a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; H_1\left(\tilde{A}_2^U\right), H_2\left(\tilde{A}_2^U\right)\right), \\ \left(a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; H_1\left(\tilde{A}_2^L\right), H_2\left(\tilde{A}_2^L\right)\right) \end{pmatrix}$$

is defined as follows [5, 20]:

$$\tilde{\tilde{A}}_1 \otimes \tilde{\tilde{A}}_2 = \left(\tilde{A}_1^U, \tilde{A}_1^L\right) \otimes \left(\tilde{A}_2^U, \tilde{A}_2^L\right) = \\ \left(\begin{array}{c} \begin{pmatrix} a_{11}^U \times a_{21}^U, a_{12}^U \times a_{22}^U, a_{13}^U \times a_{23}^U, a_{14}^U \times a_{24}^U; \\ \min\left(H_1\left(\tilde{A}_1^U\right), H_1\left(\tilde{A}_2^U\right)\right) \min\left(H_2\left(\tilde{A}_1^U\right), H_2\left(\tilde{A}_2^U\right)\right) \end{pmatrix}, \\ \begin{pmatrix} a_{11}^L \times a_{21}^L, a_{12}^L \times a_{22}^L, a_{13}^L \times a_{23}^L, a_{14}^L \times a_{24}^L; \\ \min\left(H_1\left(\tilde{A}_1^L\right), H_1\left(\tilde{A}_2^L\right)\right), \min\left(H_2\left(\tilde{A}_1^L\right), H_2\left(\tilde{A}_2^L\right)\right) \end{pmatrix} \end{array} \right) \tag{10}$$

Definition 3.5. *Some arithmetic operations between the trapezoidal interval type-2 fuzzy set*

$$\tilde{\tilde{A}}_1 = \left(\tilde{A}_1^U, \tilde{A}_1^L\right) = \begin{pmatrix} \left(a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1\left(\tilde{A}_1^U\right), H_2\left(\tilde{A}_1^U\right)\right), \\ \left(a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; H_1\left(\tilde{A}_1^L\right), H_2\left(\tilde{A}_1^L\right)\right) \end{pmatrix}$$

and the crisp value k are defined as follows [20]:

$$k \times \tilde{\tilde{A}}_1 = \begin{pmatrix} \left(k \times a_{11}^U, k \times a_{12}^U, k \times a_{13}^U, k \times a_{14}^U; H_1\left(\tilde{A}_1^U\right), H_2\left(\tilde{A}_1^U\right)\right), \\ \left(k \times a_{11}^L, k \times a_{12}^L, k \times a_{13}^L, k \times a_{14}^L; H_1\left(\tilde{A}_1^L\right), H_2\left(\tilde{A}_1^L\right)\right) \end{pmatrix} \quad (11)$$

$$\frac{\tilde{\tilde{A}}_1}{k} = \begin{pmatrix} \left(\frac{1}{k} \times a_{11}^U, \frac{1}{k} \times a_{12}^U, \frac{1}{k} \times a_{13}^U, \frac{1}{k} \times a_{14}^U; H_1\left(\tilde{A}_1^U\right), H_2\left(\tilde{A}_1^U\right)\right), \\ \left(\frac{1}{k} \times a_{11}^L, \frac{1}{k} \times a_{12}^L, \frac{1}{k} \times a_{13}^L, \frac{1}{k} \times a_{14}^L; H_1\left(\tilde{A}_1^L\right), H_2\left(\tilde{A}_1^L\right)\right) \end{pmatrix} \quad (12)$$

Definition 3.6. *The ranking value $Rank\left(\tilde{\tilde{A}}_i\right)$ of the trapezoidal interval type-2 fuzzy set $\tilde{\tilde{A}}_i$ is defined as follows [5, 20]:*

$$\begin{aligned} Rank\left(\tilde{\tilde{A}}_i\right) &= M_1\left(\tilde{A}_i^U\right) + M_1\left(\tilde{A}_i^L\right) + M_2\left(\tilde{A}_i^U\right) + M_2\left(\tilde{A}_i^L\right) + M_3\left(\tilde{A}_i^U\right) \\ &+ M_3\left(\tilde{A}_i^L\right) - \frac{1}{4}\left(S_1\left(\tilde{A}_i^U\right) + S_1\left(\tilde{A}_i^L\right) + S_2\left(\tilde{A}_i^U\right) + S_2\left(\tilde{A}_i^L\right)\right) \\ &- \frac{1}{4}\left(S_3\left(\tilde{A}_i^U\right) + S_3\left(\tilde{A}_i^L\right) + S_4\left(\tilde{A}_i^U\right) + S_4\left(\tilde{A}_i^L\right)\right) + H_1\left(\tilde{A}_i^U\right) \\ &+ H_1\left(\tilde{A}_i^L\right) + H_2\left(\tilde{A}_i^U\right) + H_2\left(\tilde{A}_i^L\right) \end{aligned} \quad (13)$$

where $M_p(\tilde{A}_i^j)$ denotes the average of the elements a_{ip}^j and $a_{i(p+1)}^j$,

$$M_p\left(\tilde{A}_i^j\right) = \left(a_{ip}^j + a_{i(p+1)}^j\right)/2, \quad \leq p \leq 3 \quad (14)$$

denotes the standard deviation of the elements $a_{i1}^j, a_{i2}^j, a_{i3}^j, a_{i4}^j$

$$S_q\left(\tilde{A}_i^j\right) = \sqrt{\frac{1}{2}\sum_{k=q}^{q+1}\left(a_{ik}^j - \frac{1}{2}\sum_{k=q}^{q+1} a_{ik}^j\right)}, \quad 1 \leq q \leq 3 \quad (15)$$

$S_4\left(\tilde{A}_i^j\right)$ *denotes the standard deviation of the elements* $a_{i1}^j, a_{i2}^j, a_{i3}^j, a_{i4}^j$

$$S_4\left(\tilde{A}_i^j\right) = \sqrt{\frac{1}{4}\sum_{k=1}^{4}\left(a_{ik}^j - \frac{1}{4}\sum_{k=1}^{4} a_{ik}^j\right)^2} \tag{16}$$

$H_p(\tilde{A}_i^j)$ *denotes the membership value of the element* $a_{i(p+1)}^j$ *in the trapezoidal membership function* $\tilde{A}_i^j$, $1 \le p \le 2$, $j \in \{U, L\}$ *and* $1 \le i \le n$.

4 TYPE REDUCTION FOR TYPE-2 FUZZY SETS

The output of a type-1 fuzzy logic system is a type-1 fuzzy set. This set is well known so there are many defuzzification methods to obtain crisp numbers. A short time results because type-2 fuzzy logic systems have been developed, and the output is a type-2 fuzzy set. A type reduction method in a type-2 fuzzy set is an important step. The aim of the type reduction process is to convert an interval type-2 fuzzy set into a type-1 fuzzy set [21].

4.1 Centroid of a Type-2 Fuzzy Set

$C_{\tilde{\tilde{A}}}$ is the centroid of an interval type-2 fuzzy set $\tilde{\tilde{A}}$.

$$C_{\tilde{\tilde{A}}} = \frac{1}{[C_l, C_r]} \tag{17}$$

where C_l and C_r are the minimum and maximum points of centroid $\tilde{\tilde{A}}$, respectively. These numbers exist because the centroid of each of the embedded type-1 fuzzy sets is a bounded number. Associated with each of these numbers is a membership grade of 1, because the secondary grades of an interval type-2 fuzzy sets are all equal to 1.

$$C_l = \min(C_{\tilde{\tilde{A}}}) = \frac{\int_{-\infty}^{L} x\overline{\mu}_{\tilde{\tilde{A}}}(x)dx + \int_{L}^{\infty} x\underline{\mu}_{\tilde{\tilde{A}}}(x)dx}{\int_{-\infty}^{L} \overline{\mu}_{\tilde{\tilde{A}}}(x)dx + \int_{L}^{\infty} \underline{\mu}_{\tilde{\tilde{A}}}(x)dx} \tag{18}$$

$$C_r = \max(C_{\tilde{\tilde{A}}}) = \frac{\int_{-\infty}^{R} x\underline{\mu}_{\tilde{\tilde{A}}}(x)dx + \int_{R}^{\infty} x\overline{\mu}_{\tilde{\tilde{A}}}(x)dx}{\int_{-\infty}^{R} \underline{\mu}_{\tilde{\tilde{A}}}(x)dx + \int_{R}^{\infty} \overline{\mu}_{\tilde{\tilde{A}}}(x)dx} \tag{19}$$

where L and R are the switch points that define the change from $\bar{\mu}_{\tilde{\tilde{A}}}(x)$ to $\underline{\mu}_{\tilde{\tilde{A}}}(x)$ and $\underline{\mu}_{\tilde{\tilde{A}}}(x)$ to$\bar{\mu}_{\tilde{\tilde{A}}}(x)$, respectively, and $C_l = L$ and $C_r = R$. To calculate (18) and (19), the L and R switch points must be known [8, 21, 22].

4.2 Type Reduction Indices Methods

For the type reduction of interval type-2 fuzzy sets, Niewiadomski *et al.* [23] offered optimistic, pessimistic, realistic and weighted average indices:

$$TR_{opt}(\tilde{\tilde{A}}) = \bar{\mu}_{\tilde{\tilde{A}}}(x), x \in X \tag{20}$$

$$TR_{pes}(\tilde{\tilde{A}}) = \underline{\mu}_{\tilde{\tilde{A}}}(x), x \in X \tag{21}$$

$$TR_{re}(\tilde{\tilde{A}}) = 0,5^*(\bar{\mu}_{\tilde{\tilde{A}}}(x) + \underline{\mu}_{\tilde{\tilde{A}}}(x)), x \in X \tag{22}$$

$$TR_{wa}(\tilde{\tilde{A}}) = w_1\underline{\mu}_{\tilde{\tilde{A}}}(x) + w_2\bar{\mu}_{\tilde{\tilde{A}}}(x), x \in X \tag{23}$$

where w_1 and w_2 are the coefficients that satisfy $w_1 + w_2 = 1$

4.3 Modified Best Nonfuzzy Performance Methods

Kahraman *et al.* [24] offered a method for triangular and trapezoidal type-2 fuzzy sets by modifying the Best Nonfuzzy Performance (BNP) value for defuzzifying and ranking interval type-2 fuzzy sets. The proposed defuzzified Triangular Type-2 Fuzzy Set (DTriT) approach follows:

$$DTriT = \frac{\frac{(u_U - l_U) + (m_U - l_U)}{3} + l_U + \alpha\left[\frac{(u_L - l_L) + (m_L - l_L)}{3} + l_L\right]}{2} \tag{24}$$

where α is the maximum membership degree of the lower membership function of the type-2 fuzzy set considered; u_U is the largest possible value of the upper membership function. l_U is the least possible value of the upper membership function. m_U is the most possible value of the upper membership function. u_L is the largest possible value of the lower membership function. l_L is the least possible value of the lower membership function and m_L is the most possible value of the lower membership function.

$$DTraT = \frac{\frac{(u_U - l_U) + (\beta_U{}^* m_{1U} - l_U) + (\alpha_U{}^* m_{2U} - l_U)}{4} + l_U}{2} + \frac{\left[\frac{(u_L - l_L) + (\beta_L{}^* m_{1L} - l_L) + (\alpha_L{}^* m_{2L} - l_L)}{4} + l_L\right]}{2} \tag{25}$$

where α and β are the maximum membership degrees of the lower membership function of the type-2 fuzzy set considered; u_U is the largest possible value of the upper membership function. l_U is the least possible value of the upper membership function. m_{1U} and m_{2U} are the second and third parameters of the upper membership function. u_L is the largest possible value of the lower membership function. l_L is the least possible value of the lower membership function and m_{1L} and m_{2L} are the second and third parameters of the lower membership function.

5 THE PROPOSED METHOD FOR TYPE-2 FUZZY ANP

In this section, we proposed an interval type-2 fuzzy ANP method for modelling vagueness originating from both the linguistic variables of experts and membership functions as follows:

5.1 Type-1 Fuzzy ANP

Decision makers use verbal expressions to compare criteria in pairwise comparisons. Experts prefer to express with verbally of their views on a topic and this will be more accurate than the use of exact number. Because there are qualitative criteria, interactions among the criteria and linguistic variables, fuzzy ANP, which is a combination of ANP and Fuzzy Logic methods was developed. Additionally, in the literature, fuzzy ANP methods are based on type-1 fuzzy sets. There are several methods that use the type-1 fuzzy ANP. Onut *et al.* [25] is based on a type-1 fuzzy ANP approach that is used for transportation-mode selection between Turkey and Germany. Dagdeviren and Yuksel [26] measured the sectoral competition level (SCL) of an organization by using type-1 fuzzy ANP technique. Li *et al.* [27] applied fuzzy ANP due to the success of with complex problems and to eliminate the uncertain judgement of decision makers'. 16 sub-criteria and 4 main criteria (availability, cost, quality and company's reputation) are described in this research. Guneri *et al.* [28] been applied the fuzzy ANP method for selecting a shipyard location. Chiang and Tzeng [29], Ayag and Ozdemir [30], Kumar and Maiti [31], and Binici *et al.* [32] applied type-1 fuzzy ANP for the selection of the best 3PL company to resolve dynamic and uncertain environments chosen to the method. This research was developed by using Chang's Extent Analysis. Abdullah and Najib [33] proposed a new fuzzy AHP characterized by interval type-2 fuzzy sets by using the likelihood approach of Chen and Lee [5, 20]. Buckley [3] developed another extension of the method from Saaty' s AHP [34] method with a_{ij} fuzzy comparative rates. Buckley [3] called attention to two problems in Van Laarhoven and Pedrycz' s methods [1]: It is necessary to use the absolute exponential fuzzy numbers and in the absence of solution of linear equations. Buckley has used the geometric mean to calculate

Linguistic scales	Fuzzy scales
Equally important (E)	(1, 1, 1)
Weakly important (WI)	(1, 3, 5)
Strongly important (S)	(3, 5, 7)
Very strongly important (VS)	(5, 7, 9)
Absolutely important (AS)	(7, 9, 9)

TABLE 1
Definition of the fuzzy scale of the linguistic variables

performance scores and solve these problems. In this case, the only solution is guaranteed for comparison matrices. The steps of Buckley's method are given below [3].

Step 1: Establishing Model
A Network Model has been designed to handle inner dependence, outer dependence and feedback.

Step 2: Comparing Criteria and Checking the Consistency Ratio
Opinions of experts represent a triangular fuzzy number instead of an exact number in this step because of eliminating uncertainties.

$$\tilde{A} = \begin{vmatrix} 1 & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & 1 & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & 1 \end{vmatrix} = \begin{vmatrix} 1 & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ {}^{1}/_{\tilde{a}_{21}} & 1 & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ {}^{1}/_{\tilde{a}_{n1}} & {}^{1}/_{\tilde{a}_{n2}} & \cdots & 1 \end{vmatrix}$$

where $\tilde{a}_{ij}$ are triangular fuzzy numbers according to *Table 1*.

To check the consistency ratio, triangular fuzzy numbers were defuzzified according to the graded mean integration method.

$$A = \frac{l + 4m + u}{6} \tag{26}$$

According to the graded mean integration approach, triangular fuzzy numbers were transformed into an exact number. If the consistency ratio is less than 0.10, the comparison is acceptable. If the comparison is not consistent, a pairwise comparison is compared again by experts.

Step 3: Calculating Fuzzy Weights
The fuzzy geometric mean for each row of matrices is calculated for the weights of criteria and alternatives.

$$\tilde{r}_i = (\tilde{a}_{i1} \otimes \ldots \otimes \tilde{a}_{in})^{1/n}, \forall i \tag{27}$$

$$\tilde{w}_i = \tilde{r}_i \otimes (\tilde{r}_1 \oplus \ldots \oplus \tilde{r}_n)^{-1} \tag{28}$$

$$\tilde{w}_i = (l_i, m_i, u_i)$$

where $\otimes$ and $\oplus$ are fuzzy multiplication and addition operations.

$$\tilde{U}_i = \sum_{\imath=1}^{n} \tilde{w}_j \tilde{r}_{ij} \tag{29}$$

where $\tilde{U}_i$ is the fuzzy utility of alternatives or criteria.

Step 4: Obtaining the Best Non-Fuzzy Performance (BNP) Number

$$BNP_{\tilde{w}_i} = \left[\frac{(u_i - l_i) + (m_i - l_i)}{3} + l_i\right], \forall i \tag{30}$$

Using *Equation 30*, defuzzified weights were calculated. Therefore, defuzzificiation is applied according to the BNP method.

Step 5: Selection the best alternative
The best alternative that has the maximum weight is selected among all of the alternatives according to the BNP number.

5.2 Type-2 Fuzzy ANP

In this section, Buckley's method will be modified by the use of interval type-2 fuzzy sets. [24]

Step 1: Establishing the Network Model
This step is applied like the type-1 Fuzzy ANP. The main criteria, sub-criteria, alternatives, inner/outer dependence and feedbacks are determined by experts.

Linguistic Variables	Trapezoidal interval fuzzy scales
Equally important (E)	(1,1,1,1;1,1) (1,1,1,1;1,1)
Weakly important (WI)	(1,2,4,5;1,1) (1.2,2.2,3.8,4.8;0.8,0.8)
Strongly important (S)	(3,4,6,7;1,1) (3.2,4.2,5.8,6.8;0.8,0.8)
Very strongly important (VS)	(5,6,8,9;1,1) (5.2,6.2,7.8,8.8; 0.8,0.8)
Absolutely important (AS)	(7,8,9,9;1,1) (7.2, 8.2, 8.8, 9; 0.8, 0.8)

TABLE 2
Definition interval type-2 fuzzy scale of the linguistic variables

Step 2: Comparing Criteria and Checking Consistency Ratio
After establishing the network, fuzzy comparison matrices are evaluated among all of the criteria of the network systems according to *Table 2.*

$$\tilde{\tilde{A}} = \begin{vmatrix} 1 & \tilde{\tilde{a}}_{12} & \dots & \tilde{\tilde{a}}_{1n} \\ \tilde{\tilde{a}}_{21} & 1 & \cdots & \tilde{\tilde{a}}_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \tilde{\tilde{a}}_{n1} & \tilde{\tilde{a}}_{n2} & \cdots & 1 \end{vmatrix} = \begin{vmatrix} 1 & \tilde{\tilde{a}}_{12} & \dots & \tilde{\tilde{a}}_{1n} \\ 1/\tilde{\tilde{a}}_{21} & 1 & \cdots & \tilde{\tilde{a}}_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ 1/\tilde{\tilde{a}}_{n1} & 1/\tilde{\tilde{a}}_{n2} & \cdots & 1 \end{vmatrix}$$

where

$$1/\tilde{\tilde{a}} = \left(\begin{array}{c} \left(\frac{1}{a_{14}^U}, \frac{1}{a_{13}^U}, \frac{1}{a_{12}^U}, \frac{1}{a_{11}^U}; H_1(a_{12}^U), H_2(a_{13}^U)\right), \\ \left(\frac{1}{a_{24}^L}, \frac{1}{a_{23}^L}, \frac{1}{a_{22}^L}, \frac{1}{a_{21}^L}; H_1(a_{22}^L), H_2(a_{23}^L)\right) \end{array} \right) \tag{31}$$

The consistency of each of the pairwise comparison matrices is checked like classical fuzzy ANP by using defuzzified matrices.

Step 3: Calculating Geometric Means and Fuzzy Weights
The geometric mean of each row is computed. Then, the fuzzy weights are calculated by normalization. The geometric mean of each row $\tilde{\tilde{r}}_i$ is calculated as

$$\tilde{\tilde{r}}_i = \left(\tilde{\tilde{a}}_{i1} \otimes \dots \otimes \tilde{\tilde{a}}_{in}\right)^{1/n}, \forall i \tag{32}$$

where

$$\sqrt[n]{\tilde{\tilde{a}}_{ij}} = \left(\begin{array}{c} \left(\sqrt[n]{a_{ij1}^U}, \sqrt[n]{a_{ij2}^U}, \sqrt[n]{a_{ij3}^U}, \sqrt[n]{a_{ij4}^U}; H_1^U(a_{ij}), H_2^U(a_{ij})\right), \\ \left(\sqrt[n]{a_{ij1}^L}, \sqrt[n]{a_{ij2}^L}, \sqrt[n]{a_{ij3}^L}, \sqrt[n]{a_{ij4}^L}; H_1^L(a_{ij}), H_2^L(a_{ij})\right) \end{array} \right)$$

The fuzzy weight of the ith criterion is computed as;

$$\tilde{\tilde{w}}_i = \tilde{\tilde{r}}_i \otimes \left(\tilde{\tilde{r}}_1 \oplus \ldots \oplus \tilde{\tilde{r}}_n\right)^{-1} \tag{33}$$

where

$$\frac{\tilde{\tilde{a}}_{ij}}{\tilde{\tilde{b}}_{ij}} = \left(\begin{array}{l} \frac{a_1^U}{b_4^U}, \frac{a_2^U}{b_3^U}, \frac{a_3^U}{b_2^U}, \frac{a_4^U}{b_1^U}, \min\left(H_1^U(a), H_1^U(b)\right), \min\left(H_2^U(a), H_2^U(b)\right); \\ \frac{a_1^L}{b_4^L}, \frac{a_2^L}{b_3^L}, \frac{a_3^L}{b_2^L}, \frac{a_4^L}{b_1^L}, \min\left(H_1^L(a), H_1^L(b)\right), \min\left(H_2^L(a), H_2^L(b)\right) \end{array} \right)$$

The fuzzy weights are obtained as follows:

$$\tilde{\tilde{U}}_i = \sum_{\imath=1}^{n} \tilde{\tilde{w}}_j \tilde{\tilde{r}}_{ij}$$

where $\tilde{\tilde{U}}_i$ is the fuzzy utility of alternative or criteria.

Step 4: Obtaining Defuzzified Weights, Supermatrices and the Limit Supermatrix

Using the proposed DTraT methods (*Equation 25)* Kahraman *et al.* [24], defuzzifications of main, sub-criteria and alternative weights are made using both inner/outer dependences and feedback.

An unweighted supermatrix that includes both inner/outer dependences and feedback is handled by using the weights obtained. The weighted supermatrix are calculated by applying a normalization of the unweighted supermatrix. The limit supermatrix is calculated by multiplying 2^{k+1} times the weighted supermatrix where k is a very large number. The limit supermatrix shows the limit weights of main, sub-criteria and alternatives.

Step 5: Selection of the best alternative

The weights of alternatives are calculated by multiplying the weights of the main criteria, the sub-criteria, the limits and the alternatives that were determined by experts. Additionally, normalization is applied for all criteria. Finally, the alternative that has the maximum weight is selected among all of the alternatives as the best alternative.

6 APPLICATION TO THE SELECTION OF A 3PL COMPANY BY USING INTERVAL TYPE-2 FUZZY ANP

In supply chain management, selection of the logistics firm is a very important multi-criteria decision making problem. Companies transfer their

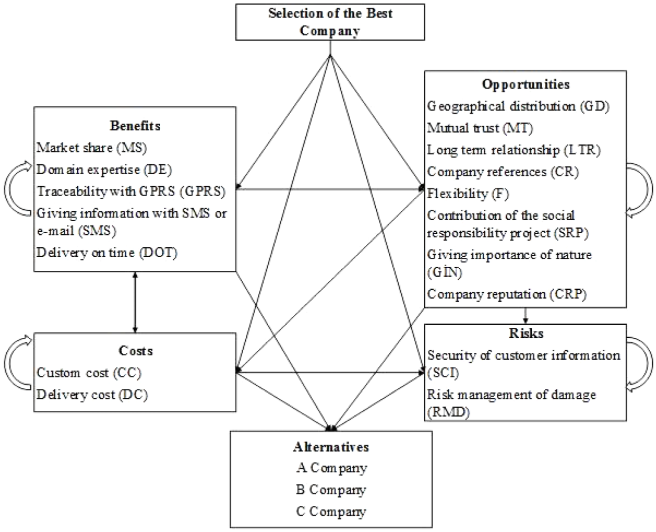

FIGURE 3
Network design of the model

logistics facilities to other companies that are experts in supply chain management to reduce costs, for quality development and to provide a competitive advantage. This event is called the 3PL. Companies must be able to effectively a use multi-criteria decision making method to compare firms, making the correct decision and preventing financial losses.

This study aims to propose a new approach by using the interval type-2 fuzzy ANP method. There are many studies of selecting a 3PL company, but there is no study that uses the interval type-2 fuzzy ANP method. Therefore, the interval type-2 fuzzy ANP method was applied in the selection of suppliers for the company that is located in Eskisehir Organized Industry in Turkey.

Step 1: Model hierarchy and network design are shown in *Figure 3* [35]. The main criteria are described under benefit, opportunities, cost and risk (BOCR) as well as according to 17 sub-criteria by expert.

Step 2: After establishing a network, expert compared the main criteria and sub-criteria by using the type-2 fuzzy scale of the linguistic variables shown in *Table 2*. The pairwise matrix of the main criteria with respect to the goal are given in *Table 3*. Additionally, the pairwise comparison matrix with type-2 fuzzy trapezoidal numbers is given in *Table 4*. In addition, the pairwise comparison matrices with type-2 fuzzy numbers of sub-criteria are handled and the pairwise comparison matrix of inner dependence with type-2 fuzzy numbers is given in *Table 5* as an example.

w.r.t. Goal	**B**	**O**	**C**	**R**
B	(1,1,1,1;1,1)(1,1,1,1;1,1)	(1, 2, 4, 5; 1, 1) (1.2, 2.2, 3.8, 4.8; 0.8, 0.8)	(1, 2, 4, 5; 1, 1) (1.2, 2.2, 3.8,4.8;0.8,0.8)	(3, 4, 6, 7; 1, 1) (3.2, 4.2,5.8, 6.8; 0.8, 0.8)
O	(0.2, 0.25, 0.5, 1; 1, 1) (0.21,0.26, 0.45, 0.83;0.8,0.8)	(1,1,1,1;1,1)(1,1,1,1;1,1)	(1, 2, 4, 5;1,1) (1.2, 2.2, 3.8, 4.8; 0.8, 0.8)	(3, 4, 6, 7; 1, 1) (3.2,4.2,5.8,6.8;0.8,0.8)
C	(0.2, 0.25, 0.5, 1; 1,1) (0.21,0.26,0.45,0.83;0.8,0.8)	(0.2,0.25,0.5,1;1,1) (0.21,0.26,0.45,0.83;0.8,0.8)	(1,1,1,1;1,1)(1,1,1,1;1,1)	(5, 6, 8, 9; 1, 1) (5.2, 6.2, 7.8, 8.8)
R	(0.14,0.17,0.25,0.33;1,1) (0.15,0.17,0.24,0.31;0.8,0.8)	(0.14,0.17,0.25,0.33;1,1) (0.15,0.17,0.24,0.31;0.8,0.8)	(0.11,0.13,0.17,0.2;1,1) (0.11,0.13,0.16,0.19;0.8,0.8)	(1,1,1,1;1,1)(1,1,1,1;1,1)

TABLE 3
Pairwise comparision matrix for main criteria

Goal	**B**	**O**	**C**	**R**
B	E	WI	WI	S
O	1/WI	E	WI	S
C	1/WI	WI	E	VS
R	1/S	1/S	1/VS	E

TABLE 4
Pairwise comparision matrix for the main criteria

Market Share	**MT**	**LTR**	**CR**
MT	E	WI	WI
LTR	1/WI	E	1/S
CR	1/WI	S	E

TABLE 5
Pairwise comparision matrix for market share

A consistency ratio check of defuzzified pairwise comparison matrices was performed. The defuzzified pairwise comparison matrix was checked for its consistency ratio and found to be smaller than 0,10. This step has to be repeated for each set of pairwise comparison matrices.

Step 3: Calculate the geometric means and type-2 fuzzy weights as follows: The geometric mean of each row is calculate using *Equation 3.* The geometric mean of the first row is calculated as;

$$
\begin{aligned}
&\tilde{\tilde{r}}_B = \left(\tilde{\tilde{a}}_{11} \otimes \tilde{\tilde{a}}_{12} \otimes \tilde{\tilde{a}}_{13} \otimes \tilde{\tilde{a}}_{14}\right)^{1/4} \\
&= [(1, 1, 1, 1; 1, 1)\,(1, 1, 1, 1; 1, 1) \otimes \\
&(1, 2, 4, 5; 1, 1)\,(1.2, 2.2, 3.8, 4.8; 0.8, 0.8) \\
&\otimes (1, 2, 4, 5; 1, 1)\,(1.2, 2.2, 3.8, 4.8; 0.8, 0.8) \\
&\otimes (3, 4, 6, 7; 1, 1)\,(3.2, 4.2, 5.8, 6.8; 0.8, 0.8)]^{1/4} \\
&= (1.32, 2, 3.13, 3.64; 1, 1)(1.46, 2.12, 3.03, 3.54; 0.8, 0.8)
\end{aligned}
$$

$$
\begin{aligned}
&\tilde{\tilde{w}}_B = \tilde{\tilde{r}}_B \otimes (\tilde{\tilde{r}}_B \oplus \tilde{\tilde{r}}_O \oplus \tilde{\tilde{r}}_C \oplus \tilde{\tilde{r}}_R)^{-1} \\
&= (1.32, 2, 3.13, 3.64; 1, 1)(1.46, 2.12, 3.03, 3.54; 0.8, 0.8) \\
&\otimes[(1.32, 2, 3.13, 3.64; 1, 1)\,(1.47, 2.12, 3.02, 3.54; 0.8, 0.8) \\
&\oplus (0.88, 1.19, 1.86, 2.43; 1, 1)\,(0.95, 1.25, 1.78, 2.28; 0.8, 0.8) \\
&\oplus (0.67, 0.78, 1.19, 1.73; 1, 1)\,(0.69, 0.81, 1.13, 1.57; 0.8, 0.8) \\
&\oplus (0.22, 0.24, 0.32, 0.39; 1; 1)\,(0.22, 0.25, 0.31, 0.37; 0.8, 0.8)]^{-1} \\
&= (1.32, 2, 3.13, 3.64; 1, 1)(1.46, 2.12, 3.03, 3.54; 0.8, 0.8) \otimes \\
&(0.12, 0.15, 0.24, 0.32; 1, 1)(0.13, 0.16, 0.23, 0.30) \\
&= (0.16, 0.31, 0.74, 1.18; 1, 1)(0.19, 0.34, 0.68, 1.07; 0.8, 0.8)
\end{aligned}
$$

B	(1.32,2,3.13,3.64;1,1)(1.46,2.12,3.03,3.54;0.8,0.8)
O	(0.88,1.19,1.86,2.43;1,1)(0.95,1.25,1.78,2.28;0.8,0.8)
C	(0.67,0.78,1.19,1.73;1,1)(0.69,0.81,1.13,1.57;0.8,0.8)
R	(0.22,0.24,0.32,0.39;1;1)(0.22,0.25,0.31,0.37;0.8,0.8)

TABLE 6
Geometric means of the pairwise comparison matrix for the main criteria

B	(0.16,0.31,0.74,1.18;1,1)(0.19,0.34,0.68,1.07;0.8,0.8)
O	(0.11,0.18,0.44,0.79;1,1)(0.12,0.20,0.40,0.69;0.8,0.8)
C	(0.08,0.12,0.28,0.56;1,1)(0.09,0.13,0.25,0.47;0.8,0.8)
R	(0.03,0.04,0.08,0.13;1,1)(0.03,0.04,0.07,0.11;0.8,0.8)

TABLE 7
Type-2 fuzzy weights for the main criteria with respect to the goal

	MS	**DE**	**GPRS**	**SMS**	**DOT**
GD	0.0000	0.1488	0.0000	0.0000	0.0000
MT	0.0833	0.0000	0.0000	0.0000	0.0000
LTR	0.0833	0.0746	0.0000	0.0000	0.0000
CR	0.0000	0.0000	0.0000	0.0000	0.0000
F	0.0000	0.0000	0.0000	0.0000	0.0000
SRP	0.0000	0.0000	0.0000	0.0000	0.0000
GIN	0.0000	0.0000	0.0000	0.0000	0.0000
CRP	0.0833	0.1100	0.0000	0.0000	0.0000

TABLE 8
The inner dependences and feedback in supermatrix for the 3pl criteria

The type-2 fuzzy geometric means and type-2 fuzzy weights of the pairwise comparison matrix for the main criteria are given in *Table 6* and *Table 7*, respectively, for the selection of the 3PL companies.

In *Table 8*, the relationship between opportunities criteria and benefits criteria in the supermatrix are only given due to the large size of the supermatrix. According to *Table 8*, the geographical distribution (GD) is impressed by domain expertise (DE) at a rate of 0.1488. As a likely choice the company reputation (CR) is impressed by market share (MS) at a rate of 0.0833.

Using *Equation 25*, defuzzificiation weights are handled. As an example,

$$DTraT_{BENEFITS} = \frac{\frac{(1.18-0.16)+(0.85*0.31-0.16)+(1*0.74-0.16)}{4}+0.16}{2} + \frac{\left[\frac{(1.07-0.19)+(0.40*0.34-0.19)+(0.80*0.68-0.19)}{4}+0.19\right]}{2} = 0.5354$$

B	0.5354
O	0.3379
C	0.2293
R	0.0610

TABLE 9
Defuzzified weights for the main criteria with respect to the goal by using the DTraT method

Alternatives	Weights
A	0.438
B	0.376
C	0.185

TABLE 10
The overall synthesized priorities for the alternatives

$$DTraT_{OPPURTUNUTIES} = \frac{\frac{(0.79-0.11)+(0.85*0.18-0.11)+(1*0.44-0.11)}{4}+0.11}{2} + \frac{\left[\frac{(0.69-0.12)+(0.40*0.20-0.12)+(0.80*0.40-0.12)}{4}+0.12\right]}{2} = 0.3379$$

$$DTRaT_{COSTS} = \frac{\frac{(0.56-0.08)+(0.85*0.12-0.08)+(1*0.28-0.08)}{4}+0.08}{2} + \frac{\left[\frac{(0.47-0.09)+(0.40*0.13-0.09)+(0.80*0.25-0.09)}{4}+0.09\right]}{2} = 0{,}2293$$

$$DTRaT_{RISKS} = \frac{\frac{(0.13-0.03)+(0.85*0.04-0.03)+(1*0.08-0.03)}{4}+0.03}{2} + \frac{\left[\frac{(0.11-0.03)+(0.40*0.04-0.03)+(0.80*0.07-0.03)}{4}+0.03\right]}{2} = 0{,}061$$

Step 4: Using *Equation 25*, defuzzification of the main criteria, sub-criteria and alternative weights is performed for both inner/outer dependences and defuzzificiation of main criteria weights is shown in *Table 9*. A supermatrix that includes both inner/outer dependences and feedback is handled by using the weights obtained. The limit supermatrix shows the limit weights of the main and sub-criteria, as shown in *Table 10*. The limit supermatrix is calculated by multiplying 2^{k+1} times the weighted supermatrix. After this calculation, the values in the stochastic-based supermatrix converge to the constant values according to the principles of Markov Chain. Additionally, the weights of alternatives are handled by the limit supermatrix, as shown in *Table 11*.

Main Criteria	Global Weights	Sub-Criteria	Local Weights	Limit Weights
BENEFITS	0.3952	Market Share (MS)	0.2832	0.051
		Domain Expertise (DE)	0.402	0.072
		Traceability with GPRS (GPRS)	0.094	0.016
		Giving information with SMS or e-mail (SMS)	0.045	0.008
		Delivery on Time (DOT)	0.174	0.032
OPPORTUNITIES	0.2496	Geographical Distribution (GD)	0.112	0.043
		Mutual Trust (MT)	0.198	0.076
		Long Term relationship (LTR)	0.196	0.075
		Company References (CR)	0.144	0.0554
		Flexibility	0.068	0.026
		Contribution of the Social Responsibility Project (SRP)	0.058	0.019
		Giving importance of the nature (GIN)	0.058	0.022
		Company Reputation (CRP)	0.170	0.065
COSTS	0.1703	Custom Costs (CC)	0.395	0.002
		Delivery Costs (DC)	0.605	0.011
RISKS	0.1846	Risk Management of damage (RMD)	0.500	0.12
		Security of customer information (SCI)	0.500	0.12

TABLE 11
Priorities for the criteria and sub-criteria

According to *Table 10,* the category of benefits criteria is the major consideration for the selection of a 3PL company. Looking at the sub-criteria, risk management of damage and the security of customer information are the most effective criteria for the firm. Then, mutual trust, long-term relationships and the company's reputation are in order an effective criteria.

Step 5: Selecting the best supplier.
Alternatives are ranked by using weighted the sum methods and are given in *Table 11*.

When the decision model includes dependences and feedback among the criteria, Supplier A has chosen the best 3PL company.

7 CONCLUSIONS

In the logistics sector, the selection of a logistics company is a crucial problem because many criteria are included in the decision. Additionally, the selection of a logistics company is defined as a multi-criteria decision making problem. In addition, experts who made their judgement by using a linguistic term fuzzy approach needed to solve this problem. While many papers have handled the fuzzy AHP and fuzzy ANP methods, the type-2 fuzzy approach provides determination of uncertainty by incorporating fuzziness for the membership functions. Interval type-2 fuzzy sets are preferred to make the calculation easier.

Although the interval type-2 fuzzy AHP method has been introduced in the literature, the interval type-2 fuzzy ANP method is first proposed for the MCDM problem in this paper. Due to the lack of use of the interval type-2 fuzzy ANP method in this field, this study is an important contribution to the literature.

After introducing the structure of the interval type-2 fuzzy ANP method, the best 3PL company was selected by using interval type-2 fuzzy ANP according to the BOCR criteria and sub-criteria. For further research, the structure of the intuitionistic fuzzy ANP can be proposed and applied to the selection of a 3PL Company.

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