Int. Journ. of Laser Science, Vol. 1, pp. 1–15
Reprints available directly from the publisher.
Photocopying permitted by license only.

Controlling and Synchronizing the Spatiotemporal Chaos of Photorefractive Ring Oscillators with Nonlinear Feedback

X. CHEN[1,2,*], X. FENG[1], Z. YAO[1] AND Z. TIAN[1]

[1]*School of Physics, Changchun University of Science and Technology, Changchun 130022, Jilin Province, China*
[2]*School of Physics, Jilin Normal University, Siping 136000, Jilin Province, China*

We demonstrate that chaos can be controlled and converted into periodic behaviour in photorefractive ring oscillator *via* nonlinear feedback and find that the period number differs on the account of the feedback strength. By increasing the feedback strength the photorefractive ring oscillator is converted into period 8 and subsequently into period 4, 2, and 1. Nonlinear feedback is suitable for both the photorefractive ring oscillator system and spatiotemporal system. Spatiotemporal chaos can be controlled into stable periodic states and stable spatial patterns if we choose suitable feedback strength. Furthermore we present the synchronization of spatiotemporal chaos in two photorefractive ring oscillator systems *via* nonlinear feedback technology. The synchronization of spatiotemporal chaos can be achieved by adjusting the feedback strength without any pre-knowledge of the dynamic system. Numerical calculation results show that weak noise has a slight impact on synchronization, so nonlinear feedback technology is suitable in practical photorefractive ring oscillator systems.

Keywords: Photorefractive ring oscillator, spatiotemporal chaos, feedback, chaos control and synchronization

1 INTRODUCTION

Optical patterns and spatiotemporal chaos are typical nonlinear optical phenomena, Firth *et al.* discovered the spatial pattern and instability in Kerr medium for the first time in 1990 [1, 2]. Oppo *et al.* [3] and Staliunas [4]

*Corresponding author: Tel: +86 (0)431 8539 8194; E-mail: chenxx@cust.edu.cn

investigated the formation and evolution of patterns in optical parametric oscillators. Since optical patterns and spatiotemporal chaos of photorefractive oscillators were observed in experiments in 1990 [5], many scholars began to investigate the pattern and spatiotemporal chaos of photorefractive oscillators by the theoretical analysis and experimental evidence [6–10].

Controlling spatiotemporal chaos and selecting pattern have been investigated in different systems [11–15]. Zhang and Shen [16] investigated the transverse pattern of laser oscillation output in a ring cavity in optical systems, controlled spatiotemporal chaos into stable patterns in the coupled map lattice (CML) system *via* local and global phase space compressions, and restricted the turbulence and spatiotemporal chaos to the desired stable patterns successfully in the laser oscillation output in a ring cavity by uniform and non-uniform phase space compressions [17]. Yue and Shen [18, 19] investigated the optical patterns, controlling and synchronizing spatiotemporal chaos of the coupled Bragg acousto-optic bistable system. Ciofinietc [20] successfully selected and stabilized unstable patterns in a CO_2 laser system by introducing intracavity spatial perturbations which break the cylindrical symmetry of the optical cavity. By using several wires instead of a single wire, square and hexagonal patterns are obtained.

The photorefractive crystal and photorefractive effect have many potential applications, such as in optical communication, holographic storage and optical amplification [21]. The spatiotemporal chaotic phenomenon is sometimes harmful, but synchronizing spatiotemporal chaotic signals are beneficial when these signals are used to secure communication and holographic storage; therefore controlling and synchronizing spatiotemporal chaos are important subjects in the features and applications of photorefractive material. Photorefractive ring oscillator is an applicable structure of researching photorefractive material, spatiotemporal phenomena such as optical vortices, periodic alteration of transverse modes, and spatiotemporal chaos were investigated in experiment and theory [22–26]. It is surprising that up to now no clear evidence of controlling and synchronizing spatiotemporal chaos in photorefractive ring oscillator has been reported.

The present work investigates the control of spatiotemporal chaos and synchronization in the photorefractive ring oscillator. The organization of the paper is as follows. Section 2 is the introduction of the photorefractive ring oscillator and nonlinear feedback system. Section 3 is the investigation of the spatiotemporal chaos control of the photorefractive ring oscillator with nonlinear feedback technology. The stable spatial pattern is realized by selecting suitable nonlinear feedback intensity; consequently, synchronizing spatiotemporal chaos is realized between drive and response systems by selecting nonlinear feedback intensity in Section 4, we analyse the influence of the random noise and find that the influence of the random noise can be decreased by increasing feedback strength. Finally, concluding remarks are given in Section 5.

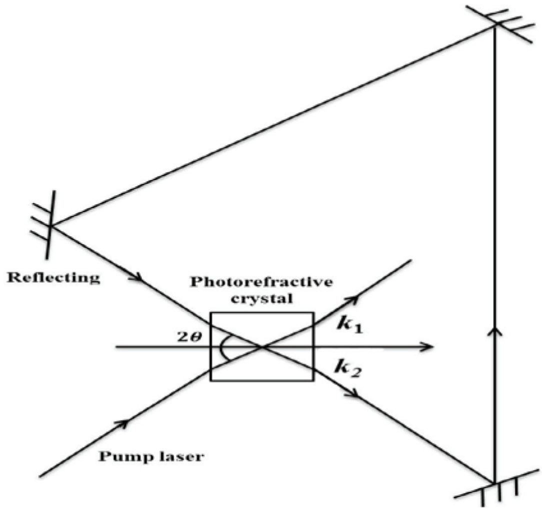

FIGURE 1
Schematic drawing of photorefractive ring oscillator.

2 THE SCHEMA OF NONLINEAR FEEDBACK CONTROL

Figure 1 shows a photorefractive ring oscillator that consists of three partially reflecting mirrors. The solid state photorefractive medium, which is pumped by an external laser beam, is inserted into the cavity. The dynamic equation of the photorefractive ring oscillator can be expressed as [27]

$$I_{n+1} = \frac{I_n\left(I_n + 1\right)}{I_n + e^{-\gamma l}} e^{-\alpha l} R\rho\left(\phi_n\right) \tag{1}$$

with

$$\rho\left(\phi_n\right) = \frac{1}{1 + F\sin^2\left(\frac{\delta + \phi_n}{2}\right)} \tag{2}$$

$$\phi_n = -\frac{\beta}{\gamma}\ln\left(\frac{I_n + 1}{I_n + e^{-\gamma l}}\right) \tag{3}$$

$$\gamma = \frac{2\pi n_1}{\lambda\cos\theta}\sin\varphi\ , \quad \beta = \frac{\pi n_1}{\lambda\cos\theta}\cos\varphi \tag{4}$$

$$n_1 = \frac{2}{\left(1+\Omega^2\tau^2\right)^{\frac{1}{2}}}\xi \tag{5}$$

and

$$\varphi = \varphi_0 + \arctan\left(\Omega\tau\right) \tag{6}$$

where I_n is signal light intensity, α is the absorption coefficient of the photorefractive crystal, R is reflectivity, l is the crystal length, $\rho(\phi_n)$ is loss due to the detuning, ϕ_n is additional phase, F is Fresnel number of the cavity, δ is cavity detuning, θ is the half-angle between the beams, $\Omega = \omega_1 - \omega_2$, τ is the decay time of the crystal, ξ is saturation value of the photo induced index change, λ is laser wavelength, φ is phase shift, φ_0 is a constant phase shift related to the nonlocal response of the crystal. When the parameters are $\delta = 0.8, \varphi_0 = \frac{\pi}{2}$, $l = 0.005$ m, $\alpha = 52$, $R = 0.92$, $\theta = 0.02$, $\lambda = 632.8$ nm, $\xi=0.00008$ and $\Omega\tau=1.25$, the photorefractive ring oscillator system is chaotic [28, 29].

The nonlinear feedback controlling system of the photorefractive ring oscillator is shown in Figure 2. According to this controlling system, the controlling dynamics is presented as

$$I_{n+1} = \left(1-k\right)I_{n+1} + kI_n \tag{7}$$

where k is the feedback strength, the photorefractive ring oscillator system is steered to different periodic orbits by varying the feedback strength through period-doubling reverse bifurcation as shown in Figure 3, from which we know when the feedback strength is in the region $0.013<k<0.015$, $0.015<k<0.029$, $0.029<k<0.155$ and $k>0.155$, the photorefractive ring oscillator system is Period 8, Period 4, Period 2 and Period 1, respectively.

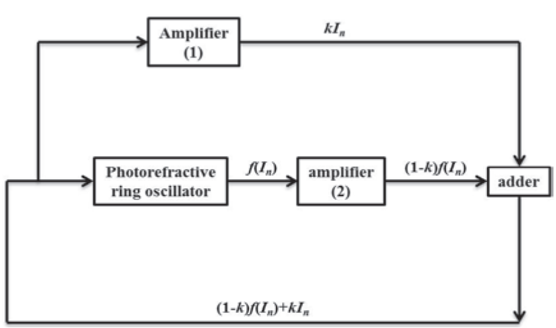

FIGURE 2
Block diagram of the nonlinear feedback controlling system of photorefractive ring oscillator.

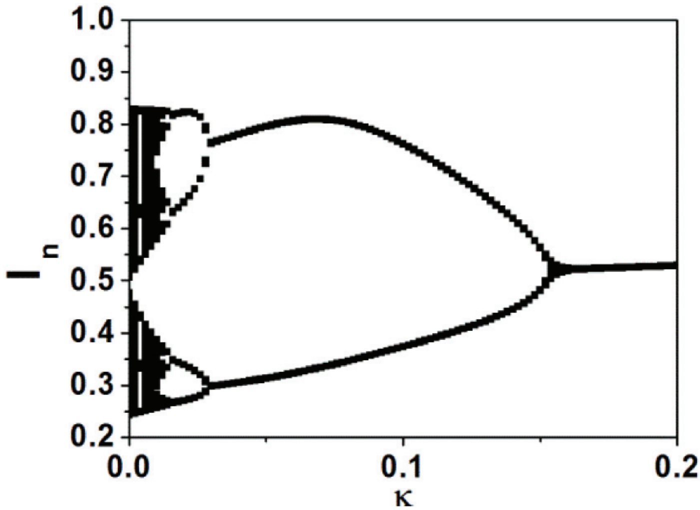

FIGURE 3
The bifurcation diagrams of I_n *versus k*.

3 CONTROLLING SPATIOTEMPORAL CHAOS AND UNSTABLE SPATIAL PATTERN WITH A NONLINEAR FEEDBACK METHOD

3.1 Controlling spatiotemporal chaos with nonlinear feedback

The space diffraction effects can lead to the space instability in the photorefractive gain medium and light field in the ring cavity interaction. Such a nonlinear optical system can be expressed using [30]

$$\frac{\partial E}{\partial t} = N\left(E\right) + iD\nabla^2 E \tag{8}$$

where $N(E)$ is a nonlinear function of light field interacting with a photorefractive crystal, E is electric field intensity, D is space diffraction coupling coefficient and ∇^2 is the transverse Laplacian. Using photon flux density parameters $I = \frac{1}{\hbar\omega}\frac{c}{8\pi}E^2 = \frac{1}{\hbar\omega}\frac{c}{8\pi}EE^*$, under the condition of slowly varying amplitude, we obtain

$$\frac{\partial I}{\partial t} = F\left(I\right) + D\left(\nabla^2 I - CI_z\right) \tag{9}$$

where $F(I)$ is a nonlinear function of light intensity, C is constant, I_z is the background noise of the light beam. I_z should be evenly distributed on the beam cross section in the ring cavity, so if we let I_z=0 and discrete

Equation (9) then we can obtain the coupled map model of light field in the ring cavity:

$$I_{n+1}(i)=(1-D)f\left(I_n(i)\right)+\frac{D}{2}\left[f\left(I_n(i-1)\right)+f\left(I_n(i+1)\right)\right] \tag{10}$$

where n=1, 2…N are the discrete steps; i=1, 2…L are the one-dimensional (1-D) lattice sites with L being the system length; and the local dynamical function $f\left(I_n(i)\right)$ is Equations (1) to (6). When the system parameters are δ=0.8, $\varphi_0=\frac{\pi}{2}$, l=0.005 m, α=52, R=0.92, θ=0.02, λ=632.8 nm, ξ=0.00008, Ωτ=1.25 and D=0.2; the initial conditions are random numbers in the interval [0, 1]; and the boundary conditions are $I_n(0)= I_n(L+1)= 0$ with L= 100, the photorefractive ring oscillator system exhibits spatiotemporally chaotic behaviours as shown in Figure 4.

Spatiotemporal chaos can be suppressed with nonlinear feedback technique in the photorefractive ring oscillatory system, the controlled dynamics under the influence of nonlinear feedback is expressed as

$$I_{n+1}(i)=(1-k)I_{n+1}(i)+kI_n(i) \tag{11}$$

Spatiotemporally chaotic behaviours are suppressed to stable periodic orbits by varying the value of k. Figures 5(a) to (c) show the local time series for the tenth lattice site, they exhibit the stabilization of the system on stable Period-8, Period-4 and Period-2 in space with k=0.023, k=0.030 and k=0.050, respectively; where the control is initiated at n=8000 and the temporal states with n<6000 are left out. Figure 5(d) shows the space-time portrait with k=0.050, from which we know that each site is in the same periodic state, but with different light intensity. Numerical calculation results prove that the maximum of the feedback strength is k=0.130. Spatiotemporal chaos of the

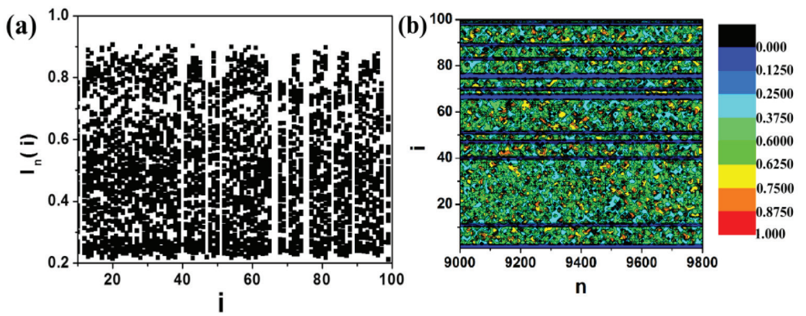

FIGURE 4
Space-time evolution of the photorefractive ring oscillator system: (a) space-amplitude plot; and (b) space-time diagram.

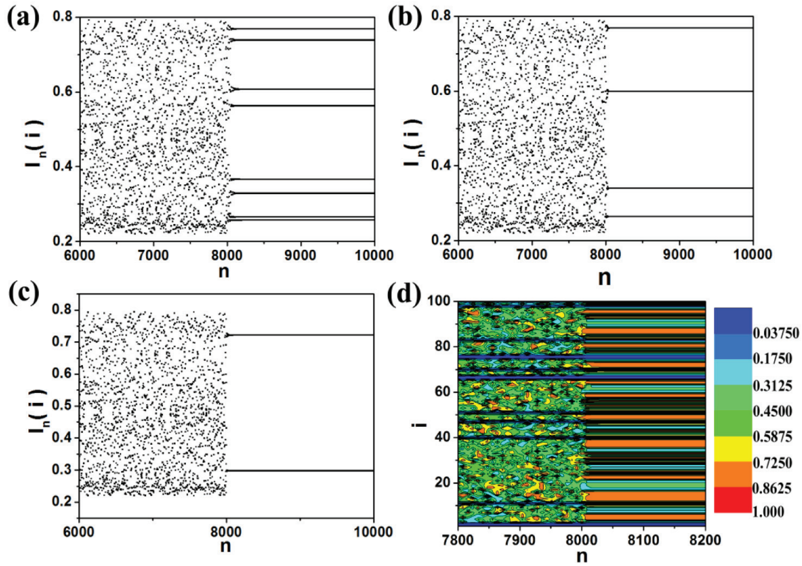

FIGURE 5
Control result of the photorefractive ring oscillator system with the nonlinear feedback, the time series of the tenth lattice site with (a) k=0.023, (b) k=0.030, (c) k=0.050 and (d) space-time diagram with k=0.050.

photorefractive ring oscillator system is not controlled into Period-1 by nonlinear feedback.

3.2 Controlling unstable spatial pattern with nonlinear feedback

We discrete Equation (9) in two-dimensional (2-D) space and get the dynamical equation of the photorefractive ring oscillator system as

$$I_{n+1}(i,j) = (1-D) f\left(I_n(i,j)\right) + \frac{D}{4}\left[f\left(I_n(i-1,j)\right) + f\left(I_n(i+1,j)\right) + f\left(I_n(i,j-1)\right) f\left(I_n(i,j+1)\right)\right] \tag{12}$$

where n=1, 2…N are the discrete steps, i, j=1, 2…L are the 2-D lattice sites. When the system parameters are δ=0.8, $\varphi_0 = \frac{\pi}{2}$, l=0.005 m, α=52, R=0.92, θ=0.02, λ=632.8 nm, ξ=0.00008, $\Omega\tau$=1.25 and D=0.2; the initial conditions are random numbers in the interval [0, 1]; and the boundary conditions are $I_n(0,j) = I_n(i,L+1) = I_n(i,0) = I_n(L+1,i)$ then the spatial pattern of the photorefractive ring oscillator system is shown in Figure 6 (we choose eight shades of Color to represent the system states).

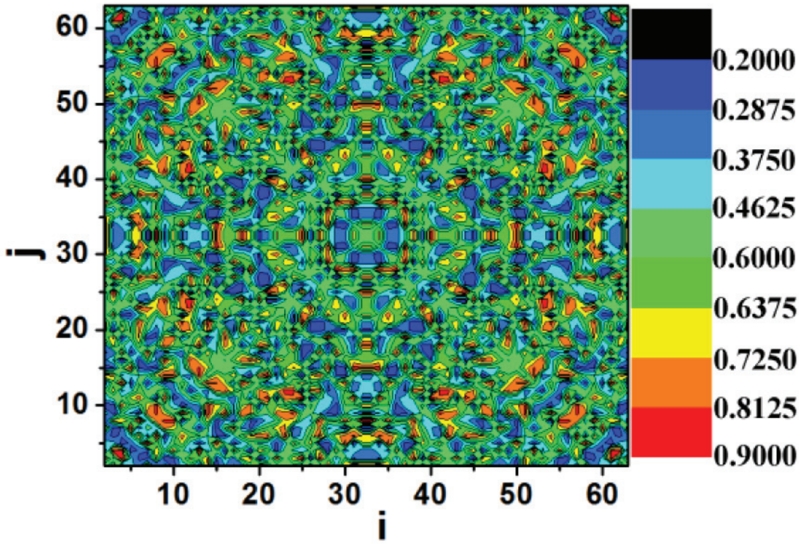

FIGURE 6
The spatial pattern of the photorefractive ring oscillator system.

The unstable spatial pattern of the photorefractive ring oscillator system is converted into stable spatiotemporal period under the influence of nonlinear feedback. The controlled dynamic equation may be written as

$$I_{n+1}(i,j) = (1-k)I_{n+1}(i,j) + kI_n(i,j) \tag{13}$$

The unstable spatial pattern as shown in Figure 6 is converted into stable spatiotemporal period by varying k. Figure 7(a) shows the local time series for the lattice coordinate (32, 32), from which we know the system is Period-2 in the time domain, where the control is initiated at n=600 with k=0.100. The spatial distribution of the optical intensity is nonuniform as shown in Figure 7(b). Increasing the feedback intensity to k=0.320, two periodic orbits gradually close to the final one as shown in Figure 7(c). When the feedback intensity is increased to k=0.350 the spatial distribution of the optical intensity is uniform except edge regions as shown in Figure 7(d).

4 SYNCHRONIZATION OF SPATIOTEMPORAL CHAOS IN THE PHOTOREFRACTIVE RING OSCILLATOR SYSTEMS

4.1 Synchronization of spatiotemporal chaos in one-dimensional (1-D) space

The schematic diagram of nonlinear feedback synchronization between two photorefractive ring oscillator systems is shown in Figure 8. We consider

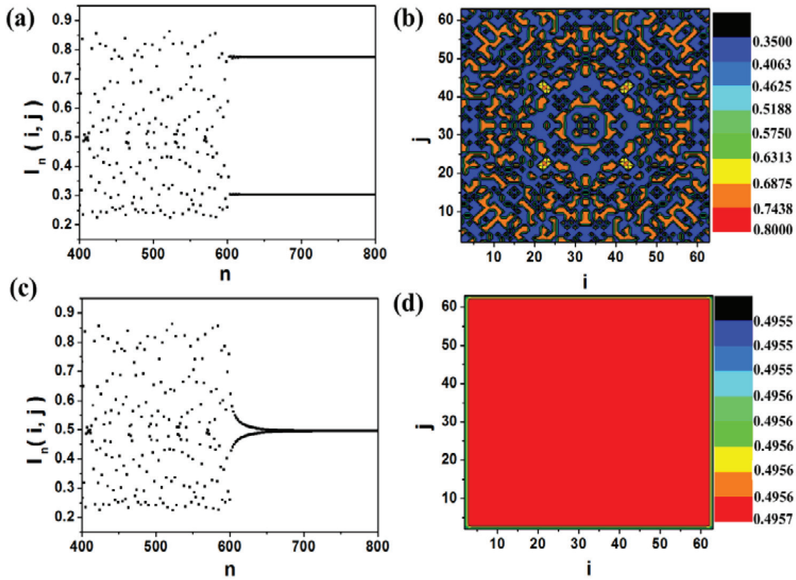

FIGURE 7
The optical intensity distribution and stable spatial pattern of the photorefractive ring oscillator system for (a) k=0.100, (b) k=0.100, (c) k=0.320 and (d) k=0.400.

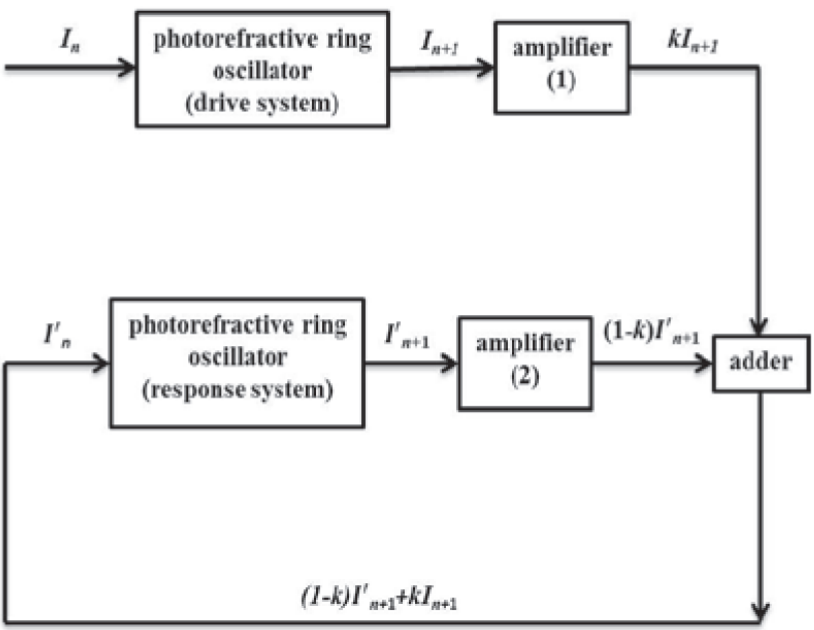

FIGURE 8
Block diagram of the nonlinear feedback synchronization.

two identical photorefractive ring oscillator systems but with different initial conditions: one of them is drive system and another is the response system. In 1-D space the drive system is described by

$$I_{n+1}(i)=(1-D)f(I_n(i))+\frac{D}{2}\left[f(I_n(i-1))+f(I_n(i+1))\right] \tag{14}$$

and the response system by

$$I'_{n+1}(i)=(1-D)f(I'_n(i))+\frac{D}{2}\left[f(I'_n(i-1))+f(I'_n(i+1))\right] \tag{15}$$

where $I_n(i)$ denotes the drive system and $I'_n(i)$ the response system. We take the nonlinear feedback method as

$$I'_{n+1}(i)=(1-k)I'_{n+1}(i)+kI_{n+1}(i) \tag{16}$$

The initial conditions of the drive system and response system are 0.20 and 0.40, respectively. The other parameters are the same as above. The space-time portraits of the drive system and response system are shown in Figure 9(a) and Figure 9(b), respectively. When k>0.340 the drive system and response system achieve synchronization as shown in Figure 9(c), the feedback is initiated at n=9000 and the temporal states with n<8000 are left out, the difference $I_{n+1}(i)-I'_{n+1}(i)$ gradually decreases and final equal to zero, indicating successful synchronization between the drive system and response system. Figure 9(d) is the time series of the response system after synchronization, from which we know the response system is still chaos.

In order to check the validity of this method, the noise term, $\vartheta\Delta\wp$, is added in the synchronization systems. Here we embed $\Delta\wp$ in the response system:

$$I'_{n+1}(i)=(1-D)f(I'_n(i))+\frac{D}{2}\left[f(I'_n(i-1))+f(I'_n(i+1))\right]\pm\Delta\wp \tag{17}$$

The random noise is in the range (0, 0.005). Here we denote the synchronization error, e, to be

$$e=\frac{I'_{n+1}(i)-I_{n+1}(i)}{I_{\max}} \tag{18}$$

Figure 9(e) shows that synchronization error is less than 6% as k=0.350. The synchronization error can be decreased by increasing feedback strength, Figure 9(f) shows that the identity synchronization can be obtained as k=0.600 and $\Delta\wp$=0.005.

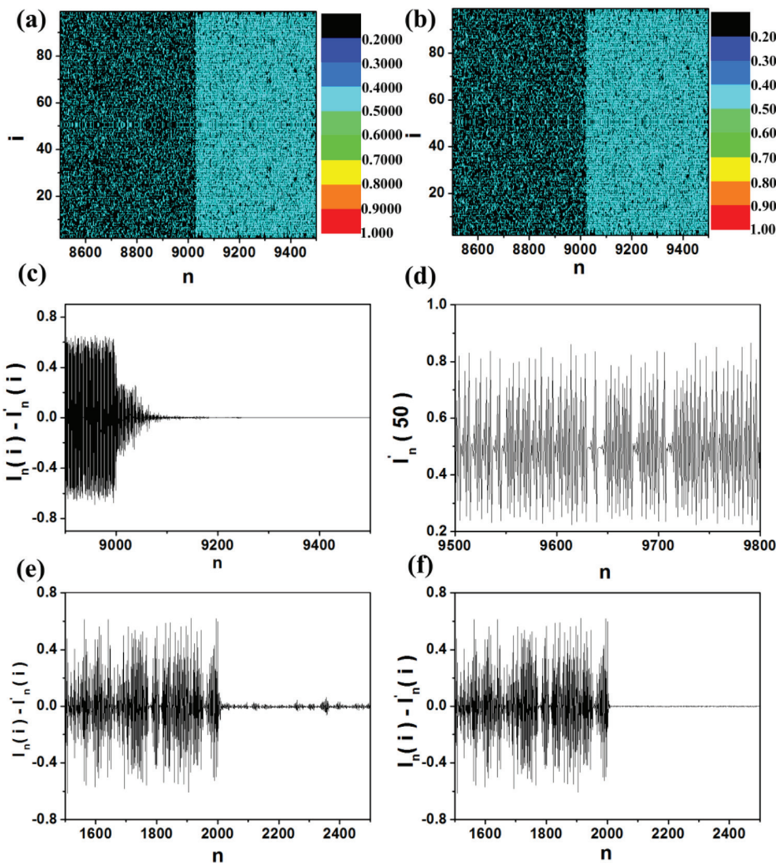

FIGURE 9
Space-time diagram for synchronizing the two photorefractive ring oscillator systems: (a) the Space-time portraits of the drive system; (b) the Space-time portraits of the response system; (c) the optical intensity difference of the drive system and drive system; (d) the time series of the response system after synchronization; (e) the optical intensity difference of the drive system and drive response; and (f) the optical intensity difference of the drive system and drive response.

4.2 Synchronization of spatiotemporal chaos in two-dimensional (2-D) space

In 2-D space the drive system is described by

$$I_{n+1}(i,j) = (1-D)f\left(I_n(i,j)\right) + \frac{D}{4}\left[f\left(I_n(i-1,j)\right) + f\left(I_n(i+1,j)\right) + f\left(I_n(i,j-1)\right) f\left(I_n(i,j+1)\right)\right] \tag{19}$$

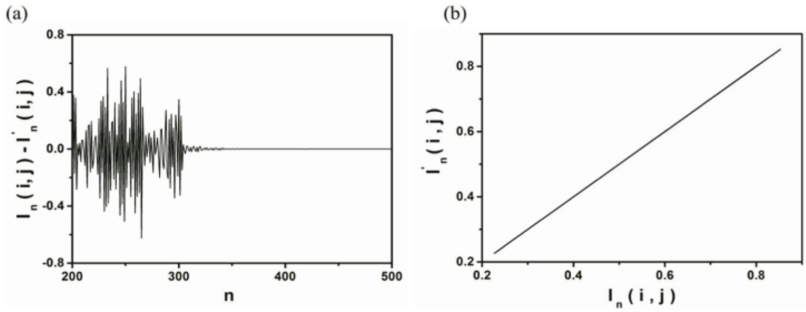

FIGURE 10
Synchronizing the two photorefractive ring oscillator systems in 1-D space: (a) the optical intensity difference of the drive system and drive response; and (b) the relationship between the drive system and response system.

and the response system by

$$I'_{n+1}(i,j) = (1-D) f\left(I'_n(i,j)\right) + \frac{D}{4}\left[f\left(I'_n(i-1,j)\right) + f\left(I'_n(i+1,j)\right) + f\left(I'_n(i,j-1)\right) f\left(I'_n(i,j+1)\right)\right] \tag{20}$$

where $I_n(i, j)$ denotes the drive system and $I'_n(i,j)$ the response system. We take the nonlinear feedback method as

$$I'_{n+1}(i,j) = (1-k) I'_{n+1}(i,j) + k I_{n+1}(i,j) \tag{21}$$

The initial conditions and parameters are the same as above. When k>0.340 the drive system and response system achieve synchronization as shown in Figure 10(a) and Figure 10(b), respectively. Numerical calculation results shown that the synchronization effect is similar with 1-D space.

5 CONCLUSIONS

The result of our numerical simulation demonstrates that spatiotemporal chaos in the photorefractive ring oscillator system can be suppressed to different periodic states by choosing appropriate feedback intensity with nonlinear feedback technique. The periodic state changes according to the feedback intensity. By increasing the feedback intensity, the photorefractive ring oscillator can be controlled into period states through period-doubling inverse bifurcation. Difference space lattice is in difference periodic states with the same feedback strength. When the feedback strength is large enough, all space lattice is periodic 1. In the global and local regions of the photorefractive ring oscillator system, unstable spatial pattern can be controlled into stable spatiotemporal period, only if a suitable feedback intensity is chosen. When two identical photorefractive ring oscillator systems have different initial conditions, they can achieve accurate chaotic synchronization *via* nonlinear feedback. Weak noise has little effect on synchronization. When the noise strength is less than 6%, the synchronization error can be reduced by increasing feedback strength and accurate chaotic synchronization can be achieved. Nonlinear feedback technique has many advantages, since no updated information of the system is required, it is not only suitable for one-dimensional (1-D), but also two-dimensional (2-D) plane, the parameter of the system dose not need to change, and control can be achieved easily by varying the feedback intensity. We can choose any feedback intensity through the optical coupler parts. So nonlinear feedback technology is suitable in practical photorefractive system.

ACKNOWLEDGMENTS

This work was funded by the National Science Foundation of Jilin province (Grant No. 20101510) and the Education Department of Jilin Province, China (Grant No. 2014487).

NOMENCLATURE

D	Space diffraction coupling coefficient, <1
E	Electric field intensity (N/C)
F	Fresnel number of the cavity
I_n	Signal light intensity
I_z	Background noise of the light beam
k	Feedback strength
l	Crystal length (m)
R	Reflectivity, <1

Greek symbols

α	Photorefractive crystal absorption coefficient (1/cm)
$\Delta\wp$	Noise term
δ	Cavity detuning
ϕ_n	Additional phase (rad)
φ	Phase shift (rad)
φ_0	Constant phase shift related to the nonlocal response of the crystal (rad)
λ	Laser wavelength (nm)
θ	Half-angle between the beams (rad)
$\rho(\phi_n)$	Loss due to the detuning
τ	Decay time of the crystal (seconds)

Ω	Frequency difference (Hz)
ξ	Saturation value of the photo-induced index change

Mathematical operators

∇^2	Transverse Laplacian.

REFERENCES

[1] Firth W.J., Fitzgerald A. and Paré. C. Transverse instabilities due to counter propagation in Kerr media. *Journal of the Optical Society of America B: Optical Physics* **7**(6) (1990), 1087–1097.

[2] Alessandro G.D. and Firth W.J. Spontaneous hexagon formation in a nonlinear optical medium with feedback mirror. *Physical Review Letters* **66**(20) (1991), 2597–2600.

[3] Oppo G.L., Brambilla M. and Lugiato L.A. Formation and evolution of roll patterns in optical parametric oscillators. *Physical Review A* **49**(3) (1994), 2028–2032.

[4] Staliunas K. Transverse pattern formation in optical parametric oscillators. *Journal of Modern Optics* **42**(6) (1995), 1261–1269.

[5] Arecchi T., Giacomelli G., Ramazza P.L. and Residori S. Experimental evidence of chaotic itinerancy and spatiotemporal chaos in optics. *Physical Review Letters* **65**(20) (1990), 2531–2534.

[6] Zhuang J. and Tan W.H. Cooperative frequency locking and spatiotemporal chaos in a photorefractive oscillator. *Journal of the Optical Society of America B: Optical Physics* **15**(8) (1998), 2249–2254.

[7] Anderson D.Z., Brockett R.W. and Nuttall N. Information dynamics of photorefractive two-beam coupling. *Physical Review Letters* **82**(7) (1999), 1418–1424.

[8] Dimmock J.O., Banerjee P.P., Madarasz F.L. and Kukhtarev N.V. Steady state analysis of self-organization of light into a scattering ring due to induced reflection gratings in photorefractive materials. *Optics Communications* **175** (2000), 433–466.

[9] Zhuang J. and Tan W.H. Instability of a multimode oscillation in a photorefractive ring oscillator. *Physical Review A* **54**(6) (1996), 5201–5209.

[10] Calvo G.F., Sturman B., Agulló-López F. and Carrascosa M. Solitonlike beam propagation along light-induced singularity of space charge in fast photorefractive media. *Physical Review Letters* **89**(3) (2002), 033902–1.

[11] Hu G. and Qu Z.L. Controlling Spatiotemporal chaos in coupled map lattice systems. *Physical Review Letters* **72**(1) (1994), 68–71.

[12] Hu G., Qu Z.L. and He. K.F. Feedback control of chaos in spatiotemporal systems. *International Journal of Bifurcation and Chaos* **5** (1995), 901–936.

[13] Sinha S. and Gupte N. Adaptive control of spatially extended systems: targeting spatiotemporal patterns and chaos. *Physical Review E* **58**(5) (1998), 5221–5224.

[14] Sinha S. and Gupte N. Targeting spatiotemporal patterns in extended systems with multiple coexisting attractors. *Physical Review E* **64** (2001), 015203.

[15] Gade P. M. Feedback control in coupled map lattices. *Physical Review E* **57**(6) (1998), 7309–7312.

[16] Zhang X. and Shen K. Control of turbulence in a two-dimensional coupled map lattice. *Physics Letters A* **299** (2002), 159–165.

[17] Zhang X. and Shen K. Controlling spatiotemporal chaos *via* phase space compression. *Physical Review E* **63** (2001), 04621–1-04621–6.

[18] Yue L.J. and Shen K. Controlling spatiotemporal chaos of coupled bistable map lattice systems using constant bias. *Physics Letters A* **340** (2005), 170–174.

[19] Yue L.J. and Shen K. Controlling and synchronizing spatiotemporal chaos of the coupled Bragg acousto-optical bistable map system using nonlinear feedback. *Chinese Physics Letters* **14**(9) (2005), 1760–1765.

[20] Ciofini M., Labate A., Meucci R. and Wang P-Y. Experimental evidence of selection and stabilization of spatial patterns in a CO_2 laser by means of spatial perturbations. *Optics Communications* **154** (1998), 307–312.

[21] Gu C., Xu Y., Liu Y.S., Pan J.J., Zhou F. and He H. Applications of photorefractive materials in information storage, processing and communication. *Optical Materials* **23** (2003), 219–227.

[22] Grigoriev R.O., Cross M.C. and Schuster H.G. Pinning control of spatiotemporal chaos. *Physical Review Letters* **79**(15) (1997), 2795–2798.

[23] Arecchi F.T., Giacomelli G., Ramazza P.L. and Residori S. Vortices and defect statistics in two-dimensional optical chaos. *Physical Review Letters* **67**(27) (1991), 3749–3752.

[24] Hennequin D., Dambly L., Dangoisse D. and Glorieux P. Basic transverse dynamics of a photorefractive oscillator. *Journal of the Optical Society of America B: Optical Physics* **11**(4) (1994), 676–684.

[25] Malos J., Vaupel M., Staliunas K. and Weiss C.O. Dynamical structures of a photorefractive oscillator. *Physical Review A* **53**(5) (1996), 3559–3564.

[26] Anderson D.Z and Saxena R. Theory of multimode operation of a unidirectional ring oscillator having photorefractive gain weak-field limit. *Journal of the Optical Society of America B: Optical Physics* **4**(2) (1987), 164–176.

[27] Yeh P. Theory of unidirectional photorefractive ring oscillators. *Journal of the Optical Society of America B: Optical Physics* **2**(12) (1985), 1924–1928.

[28] Xu J., Zhuang J., Zhao L., Li Y.F. and Li F.M. Photorefractive oscillator coupled and transverse dynamics pattern. *Science in China Series A - Mathematics Physics Astronomy* **32**(12) (2002), 1057–1065.

[29] Chen X.X., Feng X.Q., Yao Z.H. and Tian Z.L. Controlling spatiotemporal chaos of photorefractive ring oscillator with constant bias. *Proceedings of the SPIE* **8554** (2012), pp. 19–27.

[30] Lu W., Yu D. and Harrison R.G. Control of patterns in spatiotemporal chaos in optics. *Physical Review Letters* **76**(18) (1996), 3316–3319.