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Length Measurement Based on a Polychromatic Vortex Beam

D.Q. XU[1]*, L.Y. DOU[2],* AND X.B. SONG[2]

[1]Tianjin University of Technology and Education, School of Science, Tianjin 300222, China
[2]Beijing Institute of Technology, School of Physics, Beijing 100081, China

We report a theoretical study about length measurement with polychromatic vortex beam in Mach-Zehnder (MZ) interferometer. The interference results show gear shapes as orbital angular momentum interference. According to the gear shape, we can predict the length difference explicitly. Our results suggest implicit measurement can be achieved with this method.

Keywords: Polychromatic vortex beam, Mach-Zehnder (MZ) interferometer, length measurement, interference patterns

1 INTRODUCTION

Length is a fundamental physical quantity. Length measurement is a kind of basic measurement in physics, and its precise measurement is of elementary importance in science and technology. In 1887, Michelson first proposed the use of optical interferometers for the measurement of length. Heterodyne laser Michelson interferometry is widely used in precision length measurement, but the accuracy is often limited by the unknown size of the periodic non-linearity due to imperfect separation of the two optical frequencies [1].

People have developed many kinds of methods to measure length. The main methods include transit time measurements, interferometer measurements and diffraction measurements, *etc*. In 1953, Peck and Obetzt [2] explored a reversible fringe counting method to measure length [2]. In 1995, Fercher *et al.* [3] imple-

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*Corresponding author: E-mails: xudeqin0428@126.com (D.Q. Xu), doulingyu@bit.edu.cn (L.Y. Dou)

mented backscattering spectral interferometry scheme with Spatially coherent white light sources. In 2009, Hyun *et al.* [4] achieved absolute length measurement with the frequency comb of a femtosecond laser. Based on spectrum analysis, Ley and Loudon [5] discussed the quantum limits on measurements of small changes in the length of a Fabry-Perot cavity and Ikari *et al.* [6] explored a high precision beat-length measurement with in Sagnac interferometer. These efforts were also leading to many scientific and technological advances.

Recently, D'Ambrosio *et al.* [7] demonstrated ultra-sensitive angular measurements with orbital angular momentum interference gear patterns. Based on interference pattern analysis, we describe an extensive approach to exploit the frequency vortex distribution as a direct means of wavelength modulation for absolute length metrology. The frequency distribution, $\omega(\theta)$, satisfies the relationship

$$\omega(\theta) = \omega_0 + \Delta\omega\theta / 2\pi \qquad (1)$$

where θ is the polar angle. Apparently, as this beam travels, the plane perpendicular to the beam axis is no more an equiphase surface. Actually, the beam generates a helical wave front which is a little similar to a vortex light. Although, it is hard to detect the wave front, actually unnecessary, we can also put this kind of beam into implications. Small changes in the length of the interferometry produce small changes in the Mach-Zehnder (MZ) interferometer fringe intensities. If there are two frequency vortex beams interfering, we can predict the length difference according to their interference patterns.

2 RESULTS AND DISCUSSION

A schematic diagram of a MZ interferometer is shown in Figure 1. A uniform and unity polychromatic vortex beam passes through the interferometer. The inset in Figure 1 demonstrates the frequency distribution on the cross section of the beam, which satisfies Equation (1). A thin material with a refractive index, $n(\omega)$, is inserted in one arm. The material in the interferometer can be a real material or nothing, which just causes a path difference between the two arms. Interference patterns of the interferometer can be recorded by one charge-coupled device (CCD) camera. To avoid complexity, we assume the phase difference introduced by the material in the one arm is

$$\varnothing(\theta) = c\frac{\theta}{2\pi} \qquad (2)$$

where, c is related to the refractive index and length of the material. Considering an ideal detector, we can get the intensity distribution on the CCD plane as

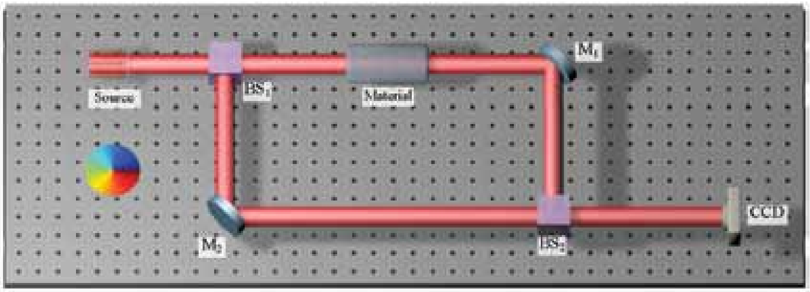

FIGURE 1
Schematic diagram of MZ interferometer. BS_1 and BS_2 are two 50/50 beam splitters, M_1 and M_2 are two mirrors. The thin material is placed in the one arm. The interferometer is illuminated by vortex polychromatic beam.

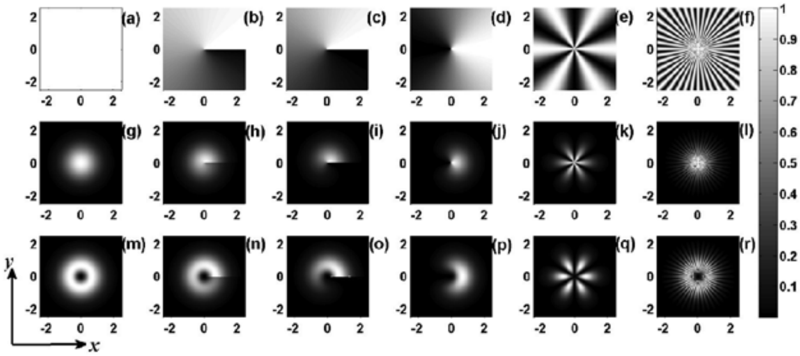

FIGURE 2
Theoretical results. The first row show the results for uniform and unity intensity sources. The second and third rows show the results for Gaussian intensity and dark hollow sources, respectively. Columns 1 to 6 from left to right correspond to the results of c =0, 0.25, 0.50, 1.00, 6.00 and 40.00 respectively.

$$I(\theta) = \sqrt{I_0\left[1 + cos\left(c\frac{\theta}{2\pi}\right)\right]} \tag{3}$$

Figure 2 demonstrates the theoretical results for polychromatic vortex beams. Because of the gradual change of the light wavelength, for simplify, we do not discuss the distribution of the light field and just analysis the intensity distribution.

First, we consider the intensity uniform and unity light sources, and the results are shown in the first row. Figures 2(a) to (f) correspond to the constant c=0, 0.25, 0.50, 1.00, 6.00 and 40.00, respectively. Figure 2(a) shows a uniform intensity distribution, which means no phase difference. Figure 2(b) and Figure 2(c) both have the fan shaped intensity profile, which means the maximum phase difference is no bigger than π. So, we have to measure the intensity values to estimate the phases. In Figure 2(d) a complete period of intensity variation is shown, which means the two arm has a 2π phase difference.

Figure 2(e) and Figure 2(f) demonstrate gear shaped patterns and we can predict the phases by counting the blades. Figure 2(e) has six bright blades, which is consistent with c=6.0. So, c=40.0 means 40 bright blazes in Figure 2(f). In the second row of Figure 2, we consider the light source has a Gaussian intensity distribution:

$$I_0\left(r\right) = exp(-r^2) \tag{4}$$

where I_0 (r) is the intensity of light source and r is the polar radius. This kind distribution is more common in laboratory setting.

Figures 2(g) to (i) correspond to c=0, 0.25, 0.50, 1.00, 6.00 and 40.00, respectively. We can also predict the phases by counting the blades like the first row.

At last, we further explore a dark hollow source to void the singularity in centre in accordance with Yin *et al.* [8] and Ni *et al.* [9]. We apply a simple style as

$$I_0 = \frac{\exp\left(-r^2\right) - exp(-r^2 / 0.25)}{max[\exp\left(-r^2\right) - \exp\left(-r^2 / 0.25\right)]} \tag{5}$$

The denominator means this source has unity value as the max intensity. The corresponding results are shown in the third row given in Figures 2(m) to (r). Except for the polar intensity distribution, we can still tell the phase difference by counting the blades.

3 CONCLUSIONS

Our scheme presented herein provides a simple and intuitive way to measure the length of a material, and also with a good resolution. Based on the Mach-Zehnder (MZ) interferometer, an application in length measurement protocol is studied. We can easily point out the length differences by counting blades of the interference gear patterns. The length measurement accuracy is increased. The resolution of our scheme can be 0.3 nm. Our results suggest implicit measurement can be achieved with this method. We hope our work can expand the applications of spectrum and remote sensing.

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