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An AP Graph Coloring Deployment Algorithm for Indoor Sensor Network Positioning

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In large indoor spaces such as complexes and logistics warehouses, sensor networks with automated mobile devices rely on high-accuracy indoor positioning systems. Due to the complexity of the indoor space structure, access point (AP) deployment is the key problem of indoor wireless location applications. The goal is to obtain an AP location scheme with high positioning accuracy and controllable deployment cost for any given indoor space structure, while the deployment algorithm itself has acceptable time overhead. This paper proposes a fast deployment algorithm based on graph theory. In this algorithm, the indoor plane space is modeled according to the coverage characteristics of wireless signals, and the problem is reduced into the k-coloring problem of weighted undirected graphs. Therefore, a graph coloring algorithm is proposed to obtain the deployment location of AP. Experimental results show that the proposed algorithm can achieve better positioning accuracy and cost deployment scheme in polynomial time compared with the existing greedy or heuristic algorithms.

Keywords: Indoor wireless location, access point, graph coloring, positioning accuracy

1 INTRODUCTION

With the development of artificial intelligence and automation technology, mobile devices such as automatic cargo robots and navigation robots are widely used in large-scale logistics warehousing, as well as complexes such

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as hospitals, shopping malls, airports and stations. These kinds of applications usually rely on indoor sensor networks with devices. Whether the sensor networks in these systems can work normally depends on high-accuracy indoor positioning services. Indoor positioning systems are mostly built based on the facilities and signals of existing wireless communication networks. For example, select WLANs (wireless local area networks), RFID (radio frequency identification), Zigbee, UWB (ultra-wide band) and other wireless signals with large coverage radii, moderate signal attenuation, wide application ranges and certain wall penetration capabilities have been used[1,2]. These positioning systems usually include wireless access points (APs) and terminal positioning equipment (e.g., a mobile sensor node). APs periodically broadcast wireless signals to announce their existence. Terminal positioning equipment receives wireless signals from an AP and obtains current location information through positioning algorithms.

The indoor positioning system for sensor networks poses new challenges to the accurate deployment of APs. The location of the AP determines the coverage of the wireless network signal in the space, which further affects the positioning accuracy. In a spatial area, any position can receive at least k AP signals, and we call the signal coverage density in this space k. In a wireless communication network, the function of the AP is to enable end users to access the network and transmit data, so when the signal coverage density k=1, the signal coverage has been completed. The indoor positioning system requires a higher signal coverage density (for most positioning algorithms, k>=3 is required). The AP deployment scheme has a significant impact on the accuracy and cost of the positioning system. Therefore, using the least number of APs in a designated space to achieve a signal coverage density of k is the key to constructing a low-cost and high-accuracy indoor positioning system.

The indoor positioning system also poses a challenge to the computing time of the AP deployment scheme. Existing research and methods have usually been oriented to the deployment of APs (or communication base stations) with signal coverage density k=1, such as in cellular networks and WLAN networks. Research on AP deployment for indoor positioning is based on greedy or heuristic algorithms (such as genetic algorithms). Due to the complexity of the indoor space pattern and wall materials, the solution space is very large, and it is difficult to converge to an approximate optimal solution in a limited time when targeting large spaces.

In this paper, the indoor space structure is mapped into a weighted undirected graph $G(V,E)$, thereby transforming the AP deployment problem into a graph coloring problem. This proves that the AP coverage for positioning is an NP problem. On this basis, this paper proposes a fast AP deployment algorithm for the positioning problem of indoor sensor networks that uses the edge cover algorithm and vertex cover algorithm for weighted undirected graphs $G(V,E)$ to iteratively obtain an approximate optimal solution. Since both the edge cover and vertex cover can obtain approximately optimal solutions in polynomial time, this algorithm is also solvable in polynomial time.

This paper is organized as follows. Section 1 introduces our research work briefly. Section 2 gives an overview about the related works. We describe the problem of AP deployment in the positioning system in Section 3. Section 4 proposes a fast deployment algorithm of AP based on graph coloring. We carry out experimental verification of the proposed method and give the major results in Section 5. Finally, we summarize our work in Section 6.

2 RELATED WORKS

Indoor location technology based on wireless signals has recently received extensive attention, and many researchers at home and abroad have conducted in-depth research. Wireless indoor location technology mainly includes two processes, namely, wireless signal acquisition and position estimation. First, in the acquisition of wireless signals, according to the physical measurement of different wireless signals, positioning methods can be divided into RSS (received signal strength), RSP (received signal phase), CSI (channel state information), TOA (time of arrival), TDOA (time difference of arrival), AOA (angle of arrival), and multi-measurement fusion [1]. Then, in the position estimation, based on different physical measurements, the target position calculation is achieved through methods such as numerical analysis, probability statistics, parameter estimation and numerical optimization.

Location estimation algorithms are usually divided into two categories: fingerprint-based positioning and ranging-based positioning. Algorithms based on fingerprint positioning [3,4] usually need to collect a large number of location fingerprints, obtain the characteristics of wireless signals through machine learning, create a wireless map and then estimate the location of the target through the best fingerprint matching. To improve the accuracy of position estimation, recent fingerprint positioning technology used deep learning methods to train signal depth features [5-8].

For the problem in which fingerprint features are degraded by environmental changes, transfer learning is used to extract transferable fingerprint features or models [9,10], thus realizing adaptive positioning technology. In addition, a ranging positioning algorithm that does not require a priori learning generally needs to estimate the distance or orientation information between an anchor node at a known location and an unknown target location, and uses multiple distance or orientation measurements to estimate the target location. In the position estimation process, according to different wireless signal measurements, the position coordinates are calculated by the trilateral positioning method, multilateral centroid method or triangulation method.

Given the spatial structure and wall material, the number and location of AP deployments determine the spatial distribution characteristics of the signal. This plays a vital role in positioning accuracy. Bhasker E. [11] proposed the ActiveCampus system, which obtains the position of the AP required for accurate positioning. When the position of the AP is inaccurate or the position of the AP changes, it affects the positioning accuracy of the system. Therefore, the system relies on user feedback to realize the accurate positioning of the AP, thereby improving the positioning accuracy of the system. He S. [12] proposed the Chameleon positioning system, which uses crowdsourced data uploaded by users to correct the changed AP information or remove the problematic AP, thereby realizing updates to the radio map and ensuring the positioning accuracy of the positioning system. Zhuang Y. [13] used crowdsourced data to estimate the parameters of an indoor signal propagation model, using the trilateral positioning method to achieve AP position estimation in the positioning system. Garcia M. [14,15] proposed wireless sensors self-location model by two approaches based on the Received Signal Strength Indicator (RSSI). The first approach uses a training session and the position is based on Neuronal Networks using the training measurements. The second approach uses triangulation model with some fixed access points. Lloret J. [16] proposed the model which is based on a derivation of the free field propagation equation taking into account the building structure and its materials. Zhang Q and He T [17] proposed a novel coverage-enhancing algorithm for hybrid wireless sensor networks. Wu H. [18] proposed a distributed range-free localization algorithm based on optimum distance derivation for three-dimensional wireless sensor networks (WSNs) in which anchor deployment is very sparse and communication range of each sensor is highly limited. Rathore R S. [19] presented a multi-hop route selection algorithm in which security is based on the newly designed multi-trust model which is formed by considering the multiple trust metrics. Dynamic deployment in wireless sensor networks (WSNs) to enhance network coverage and connectivity. Mohamed S M. [20] investigated this problem in WSNs using the Harmony Search (HS) optimization algorithm and proposed a family of five HS algorithms for dynamic deployment.

Most of the existing AP deployment research is for wireless communication applications, and its main goal is to solve the problem of seamless signal coverage of the entire network when building a communication network (signal coverage density k=1). For example, AP deployment of WLANs usually adopts the two methods of field investigation and computer simulation design [21]. Field investigation manually selects the AP installation location according to the on-site environment. The effect of this method is determined by the experience of the implementer, and it is difficult to achieve a better deployment effect for large and complex indoor structures. Computer simulation design finds the optimal deployment number and deployment location of APs by modeling the indoor structure.

There are many mathematical models for solving optimization. Commonly used optimization algorithms include genetic algorithms [22] and simulated annealing methods [23]. Reference [24] uses a classical genetic algorithm to make a reasonable layout of a single AP, the purpose of which is to obtain the maximum number of sampling points in the target environment. In [25], the author created an AP layout to achieve wide coverage of a wireless local area network and tested the loss of wireless signal penetrating obstacles. In [26], the author used a discrete gradient algorithm based on a path loss model to transform the layout problem into a continuous variable nonlinear programming problem. The author realized a reasonable layout of AP but did not carry out a reasonable experimental analysis.

Reference [27] proposed the multi-objective genetic algorithm (MOGA), which achieved the maximum signal coverage of the target environment, and used the signal-to-noise ratio as the objective function to optimize the positions of multiple APs at the same time to achieve the optimal layout of the APs. However, as the indoor environment area increases and the number of APs increases, the algorithm complexity of this method increases exponentially, which consumes considerable time. Kouhbor S. [28] used a new global optimization algorithm proposed by the University of Ballarat to optimize the location of the AP. This method can solve unconstrained continuous problems and does not require gradient information. Thus, this method is simple and convenient. Reference [29] realized the optimal placement of indoor base station antennas. Reference [30] proposed improving the positioning accuracy of the system by optimizing the location of the AP in an indoor environment. The author used the differential evolution method to optimize the location coordinates of the AP with the Euclidean distance of the signal in the fingerprint library as the objective function. The experimental results showed that the location optimization of AP can improve the positioning accuracy.

Reference [31] linked the location of the AP with the positioning performance, used a signal-to-noise ratio maximization algorithm, took the instability of the signal as the noise, maximized the signal, minimized the noise, and optimized the AP location. Reference [32] used a discrete mathematical model to find the optimal AP location. To build a discrete model, the target area was divided into grids. Since the AP could only be deployed in the center of the grid, it was necessary to increase the number of grids to achieve higher accuracy requirements. If the area of the space was large, the calculation amount of the model was often unacceptable. Reference [33] studied how to implement an AP adaptive layout in indoor positioning systems to improve the positioning accuracy of the system and proposed a proximity-greedy algorithm to achieve an AP adaptive layout. However, the author did not consider the loss model and obstacle factors.

Reference [34] considered how to increase the positioning accuracy by pressing the AP in the WLAN positioning system. This study proposed an optimization method for AP deployment based on a differential evolution algorithm. When the Euclidean distance of the received signal strength (RSS) array of all sampling as wis maximized, the positioning accuracy was improved. However, the model did not consider the factors of walls, doors and other obstacles.

Studies of AP deployment for indoor wireless positioning systems has become a research hotspot. Judging from the current research status at home and abroad, most of the research on AP deployment considers how to place the AP so that the entire network can carry out normal network communication. Part of the research considers how to place APs to obtain higher positioning accuracy, but solving the contradiction between the complex space environment and solving scale has always been the difficulty of AP deployment research. Quickly calculating a reasonable deployment position of APs in a complex indoor space is an important issue that needs to be solved when constructing a high-accuracy indoor positioning system.

3 PROBLEM DESCRIPTION

The AP deployment of the indoor sensor network discussed in this article has three prerequisites: (1) The AP deployment of the positioning system requires a higher coverage density than the AP deployment of the communication network (k>=3). (2) The positioning system adopts position fingerprint positioning technology to reduce the positioning error caused by the signal penetrating the wall or reflection, making the signal coverage the main factor in the positioning error. (3) Since the indoor space is usually divided into relatively closed floors, this article only considers the coverage of the signal in the plane space.

Definition 1: (AP deployment) $l_i=(x_i,y_i)$ represents the installation coordinates of the i-number AP in the plane space S. $D(n,s)=\{l_1,l_2...,l_n\}$ is called a deployment of n APs in S.

To solve the signal coverage of an AP in a small enclosed space area, it is necessary to consider the influence of the AP signal propagation distance in the air by the path propagation loss and signal penetration loss during signal transmission. Due to the complex indoor environment and structure, a universal and accurate indoor signal path propagation model cannot be given. This article will introduce the path loss model to describe the characteristics of the indoor environment:

$$p = p_0 - 10 \cdot n \cdot \lg\left(\frac{d}{d_0}\right) + \zeta \tag{1}$$

In the above formula, when the distance between the user and the AP is d, p represents the strength of the signal received by the user. When the distance between the user and the AP is d_0, p_0 represents the strength of the signal received by the user. The parameter n is the attenuation factor. ζ is a random variable that represents the sheltering coefficient of indoor shelters.

It is a complicated problem for multiple APs to cover a plane in a complex indoor environment. To simplify the coverage problem, this article makes the following assumptions: all walls in the indoor space have the same signal penetration loss, and a single-layer wall can reduce the signal propagation distance by d_w. From this, the signal propagation distance between any two points can be obtained:

$$dist(a,b) = |a-b| + m \cdot d_w \tag{2}$$

Definition 2: (k-coverage) Assuming that the effective signal coverage radius of all APs in the unobstructed area is R, $D(n,S)$ is a deployment of S. If $|\{l_i \mid \forall v \in S, dist(v,l_i) \le R\}| \ge k, l_i \in D(n,S)$ is satisfied, then $D(n,S)$ is said to be k-covered. That is, any position in S can be covered by the signals of at least k APs.

Definition 3: (Optimal k-deployment) is the deployment $D(m,S) \mid m = \min(n_1, n_2, ..., n_k)$ with the smallest number of APs among all k-covered deployments $\{D(n_1,S), D(n_2,S), ..., D(n_k,S)\}$ in the space area S.

This article only considers the coverage problem of k=3. The basic idea is to divide the plane space that needs to be deployed into a large connected area and a small enclosed space area. For large connected areas, we use equilateral triangle coverage; for small enclosed space areas, we choose part of the enclosed space to deploy APs.

The use of equilateral triangle coverage can simply achieve 3-coverage of signals in unobstructed areas. Three circles with the same radius are constructed to intersect, as shown in Figure 1(a). If any circle passes through the

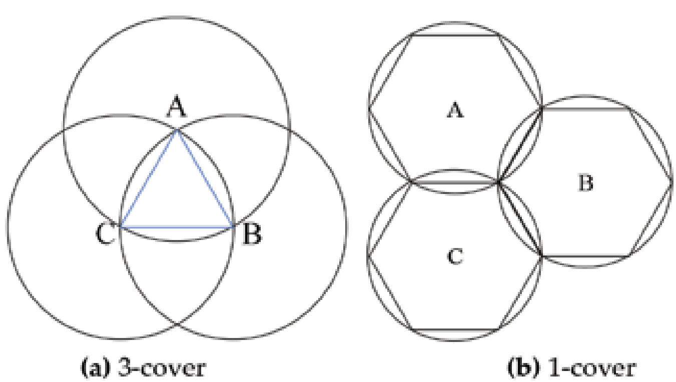


FIGURE 1
k-cover

centers of the other two circles, then the centers of the three circles form an equilateral triangle, which is covered by three circles at the same time. As long as the equilateral triangle coverage scheme is obtained, deploying APs at the vertices of the equilateral triangle can achieve 3-coverage in the plane space. The traditional cellular network used for communication only needs to achieve 1 coverage, so it only needs to cover the hexagonal space, as shown in Figure 1(b).

For small enclosed space areas, AP signal coverage is irregular. These enclosed spaces usually have a small scale. From formula 2, the main factor that determines the signal coverage is the number of walls between the two enclosed spaces. On the other hand, the positioning of the enclosed space can introduce methods such as trajectory prediction to effectively improve the accuracy. Therefore, this paper gives the following assumptions to simplify the coverage problem: Assume that all small enclosed spaces $RM = \{r_1, r_2, ..., r_j\}$ in plane S are convex polygons, and $C = \{c_1, c_2, ..., c_j\}$ is the geometric center set of each enclosed space in RM. If the geometric center c_i of a small enclosed space r_i is covered by the signal of an AP, then c_i is covered by the AP.

Therefore, the process of obtaining the AP deployment plan of the wireless positioning system in this paper is as follows: For a given plane space S, use triangular coverage to obtain the deployment scheme $L = \{l_1, l_2, ..., l_g\}$ for the large connected area in S. Next, find the smallest enclosed area deployment scheme $L_{RM} = \{l_{g+1}, l_{g+2}, ..., l_m\}$, satisfying $L_{RM} \subseteq C$, $\forall c_i \in C, | l_t \mid l_t \in L \cup L_{RM}$ and $dist(c_i, l_t) \le R$. Then, $D(m, S) = L \cup L_{RM}$ is the optimal *k*-deployment scheme of S.

4 THE DEPLOYMENT METHOD FOR 3-COVERAGE BASED ON GRAPH COLORING

To solve the 3-coverage deployment problem in indoor space, the fast AP deployment algorithm based on a weighted undirected graph proposed in this paper includes the following two main steps:

1. Modeling of indoor space plane structure. Cover the connected area with the farthest point pair greater than 2R with an equilateral triangle. The vertices of the equilateral triangle and the geometric center of the small, enclosed space are both represented by a vertex of the abstract graph. If the signal propagation distance between the two vertices does not exceed R, then an edge is constructed between the two vertices, thereby abstracting the topological relationship between all equilateral triangles and small enclosed areas as a weighted undirected graph $G(V, E)$.

2. Color the vertices of the weighted undirected graph $G(V,E)$ so that any nonedge vertex in G has at least 3 neighboring nodes to be colored. Installing an AP at the position represented by each colored vertex can make each nonedge area be covered by at least 3 AP signals. In this paper, the minimum vertex cover and edge cover algorithms are used to iteratively obtain the coloring scheme.

4.1 Spatial modeling

Initialize the vertex set V=∅ of the weighted undirected graph G(V,E). For each enclosed space in the indoor space structure plane S, if the distance between the farthest point pair of the enclosed space is $d \le 2R$, then add $V = V \cup \{v_i\}$ to represent the space to the vertex set. If $d > 2R$, then cover the enclosed space with multiple equilateral triangles with side length r, and add the vertices of each equilateral triangle to V.

As shown in Figure 2(a), there are 4 enclosed spaces r_1, r_2, r_3 and r_4 in S. Among them, the distance of the farthest point pair of r_1, r_2 and r_3 does not exceed 2R, so the nodes r_1, r_2 and r_3 corresponding to the room are generated. The farthest point pair of r_4 is larger than the coverage radius of AP signal 2R. Therefore, the room is covered by an equilateral triangle, and v is generated for each vertex of the equilateral triangle. Therefore, the vertex set $V = \{v_1, v_2, ..., v_{12}\}$ can be obtained.

Next, we initialize the edge set E=∅. For any two vertices $v_i, v_j \in R$, if $dist(v_i, v_j) \le R$, then add edge $E = E \cup \{e(v_i, v_j)\}$. The weight of $e(v_i, v_j)$, $w(v_i, v_j) = dist(v_i, v_j)$.

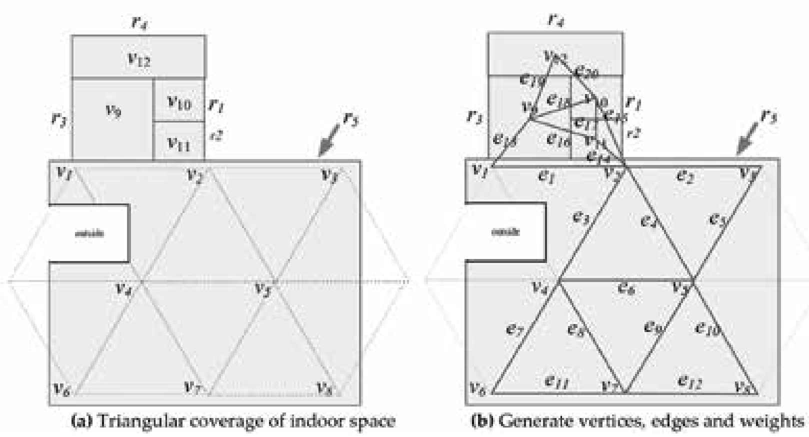


FIGURE 2
Generating a weighted undirected graph in a plane space

4.2 Graph coloring algorithm

Step 1. Initialize all vertices in vertex set V of graph G(V,E) to be colorless, and initialize the weight of each vertex v to $c(v) = 0$. Then, dye all the vertices generated by the equilateral triangles in V to red, as shown in Figure 3. The red points are marked by the lined circle ⊗.

Step 2. Initialize $G'(V', E') = G(V, E)$. For each $v' \in V'$, set the number of neighbor vertices of v' to $d(v')$ and the number of red vertices in v' itself and all its neighbor nodes to $c(v')$. If $c(v') \geq 3$ or $c(v') = d(v') + 1$, then delete vertex v' from V', and delete the edge associated with vertex v'. Next, delete the edge between the red vertices in G', as shown in Figure 4.

Step 3. Use the edge covering algorithm for the subgraph $G'(V', E')$ to obtain the minimum edge covering set E''. Delete the edges not in E'' from G', as shown in Figure 5(a). $E'' = \{e_{14}, e_{18}, e_{21}\}$) is the minimum edge cover of graph G'. Then, the vertex cover algorithm with constraints on G' is used to obtain the minimum vertex cover set V'', and the constraint is that V'' cannot contain red vertices. As shown in Figure 5(b), v_1 cannot be added to the minimum vertex cover set, so $V'' = \{v_9, v_{10}\}$. Finally, for each vertex in V'',

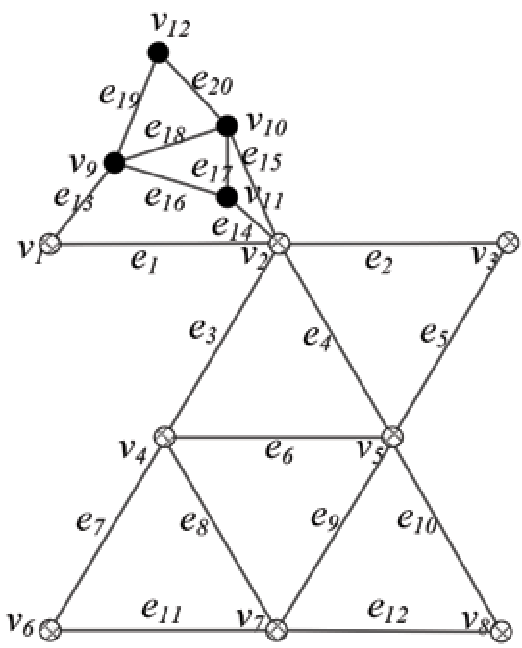


FIGURE 3
Initial coloring of weighted undirected graph

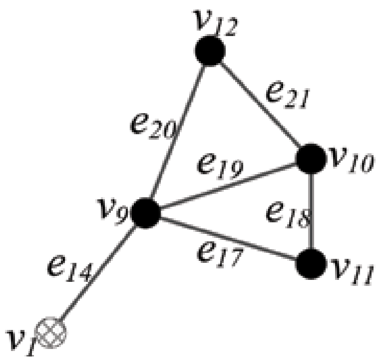
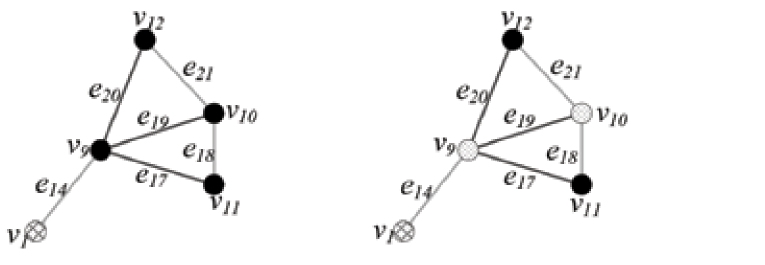


FIGURE 4
Delete the colored vertices whose neighbors have been all colored



(a) Minimum edge coverage **(b)** Minimum vertex cover with constraints

FIGURE 5
One iteration of minimum edge cover and minimum vertex cover

the corresponding vertex in $G(V,E)$ is dyed red. [To distinguish the original red vertex, the newly dyed vertex in Figure 5(b) is shown as a dotted circle].

Step 4. Repeat step 2 and step 3 until the vertex set of $G^{'}$ is empty in step 2.

The coloring scheme in Figure $G(V,E)$ can be directly converted to the AP deployment scheme, and the spatial position represented by the red vertex is the AP deployment position.

4.3 Minimum edge cover and minimum vertex cover with constraints

In step 3 of the above algorithm, it is required to obtain the minimum edge coverage of G^' (V^', E^'). First, we transform $G^{'}$ into a bipartite graph $G_b^{'}\left(V_b^{'},E_b^{'}\right)$, where $V_b^{'}$ is divided into two subsets of X and Y. For any vertex, $v_i \in V_b^{'}$ has a corresponding copy of vx_i and vy_i in X and Y. For any edge $e\left(v_i,v_j\right)\in E^{'}$, $E_b^{'} = E_b^{'} \cup e\left(vx_i,vy_j\right) \cup e\left(vy_i,vx_j\right)$.

The number of vertices in $G^{'}\left(V^{'},E^{'}\right)$ is $|V^{'}|$, and the number of vertices in the constructed bipartite graph is $|V_b^{'}|=2\times|V^{'}|$. The minimum edge coverage of the undirected graph $G^{'}$ is $|V^{'}|-C/2$, where C is the maximum matching number of the bipartite graph. Therefore, the minimum edge coverage problem is transformed into a maximum matching problem of bipartite graphs. The maximum matching problem of bipartite graphs can be obtained by the classic Hungarian algorithm.

In step 3 of Section 4.2, the minimum vertex cover $V^{"}$ with constraints of the undirected graph $G^{'}\left(V^{'},E^{'}\right)$ needs to be required. Let $V_1^{"},V_2^{"},...,V_t^{"}$ be the undirected graph $G^{'}\left(V^{'},E^{"}\right)$ covered by all vertices that are not red. Then, $|V^{"}|=\min\left(|V_1^{"}|,|V_2^{"}|,...,|V_t^{"}|\right)$. Finding the minimum vertex cover is an NP problem. This paper uses a combination of branching and greedy algorithms to calculate the approximate optimal solution.

Finally, we analyze the time complexity of the coloring algorithm in Section 4.2. First, the time complexity of the Hungary algorithm used to solve the minimum edge cover is $O\left(nm\right)$, where n is the number of vertices in the graph, and m is the number of edges in the graph. Since the vertices in $G^{'}$ whose own and all neighboring nodes are red are deleted in step 2 of the algorithm, the vertices in the nonedge regions of the large connected space covered by equilateral triangles will inevitably be deleted from $G^{'}$. Only the small space and the part at the junction of the large space and the small space are left in $G^{'}$, and these spaces are usually separated by walls. The signal used for indoor positioning usually attenuates quickly and does not have strong penetrating power, making it difficult to pass through two or more walls. Therefore, the upper limit of the number of enclosed spaces around a small enclosed space in the plane can be set to a constant q, with $m\le\frac{qn}{2}$, so that the time complexity of the minimum edge coverage is $O\left(n^2\right)$.

The time complexity of the minimum vertex cover is $O\left(n^2\log n\right)$. Therefore, the time complexity of one iteration of the coloring algorithm is $O\left(n^2\log n\right)$. This paper considers the 3-cover problem of the signal. Then, 3|V| edges are required to be matched, and at least $n_i/2$ edges are matched in the i-th iteration, where n_i represents the number of remaining vertices to be processed in the i-th iteration. Therefore, the time complexity of the algorithm described in this article is

$$O(n^2\log n+\frac{1}{4}n^2(\log n-\log 2)+\frac{1}{16}n^2(\log n-\log 4)+...)=O(n^2\log n) \quad (3)$$

Therefore, the approximate optimal solution can be obtained in polynomial time.

5 EXPERIMENTAL VERIFICATION OF AP DEPLOYMENT ALGORITHM

This article evaluates the AP deployment algorithm described in this article using three indicators: time cost of the algorithm, signal coverage of the deployment scheme, and average positioning accuracy of the positioning system after deployment. Among them, the time cost of the algorithm and the signal coverage of the deployment scheme require a large number of plane space samples, so this article uses simulation for verification. The average positioning accuracy of the positioning system is deployed and measured on the second floor of the Library Building of Zhejiang Gongshang University as a real environment site.

To ensure the diversity of the plane space structure in the simulation experiment, we use two methods to randomly generate different plane layouts. The first method first divides a large rectangular space into $n \times m$ space units with a side length of 4. There are internal walls between the units. For each internal wall, there is a probability α to open it. Then, the large space is divided into several interconnected spaces. Adjusting the parameter α can determine the degree of openness of the space. The smaller the α, the smaller enclosed spaces there are; and the larger the α, the more connected large spaces there are. The second method consists of 4×4 small rectangles and $a \times b$ large rectangles randomly forming an irregular spatial shape, where $4 << a, b << n, m$.

First, the small rectangles are randomly distributed in the plane area of the specified $n \times m$ until all the small rectangles are connected. Then, the large rectangles are randomly distributed until the ratio of the area covered by the large rectangles to the total area covered by the large and small rectangles reaches the threshold α. Obviously, the larger the α, the larger the proportion of large-scale space, and the higher the openness of the entire space. For simplicity, assuming that the AP's signal coverage distance in an open space is a constant d=15 meters, a single-layer wall can reduce the signal coverage by d_w=5 meters. For the AP signal to have the most suitable receiving intensity interval for positioning, above -90 dB is considered effective coverage.

To better evaluate the deployment results, this paper uses the deployment methods of equilateral triangle uniform coverage, square uniform coverage, equilateral triangle and full room coverage as a comparison. Uniform coverage of equilateral triangles and squares simply adopts equilateral triangles or squares with side length r to cover the entire plane space and installs an AP on the vertex of each square or equilateral triangle. Equilateral triangles and room full coverage deployment adopt equilateral triangles for uniform coverage in large connected areas, and 1 AP is installed in each area in small enclosed areas.

Figure 6(a) shows the execution time of the deployment algorithm when $\alpha = 0.5$. As the plane area to be deployed increases, the time required for deployment calculation increases. For simplification, this article ignores other terms of the time complexity polynomial and uses $F_{(X)} = kn^2 \log n$ for fitting. When k=-6.28E-09, it has a higher goodness of fit (R^2=0.9983). Since $\log n$ is close to a constant coefficient when the scale of n is limited, the calculation time of this algorithm is close to $O\left(n^2\right)$ in indoor space. Figure 6(b) shows the relationship between the execution time of the deployment algo-

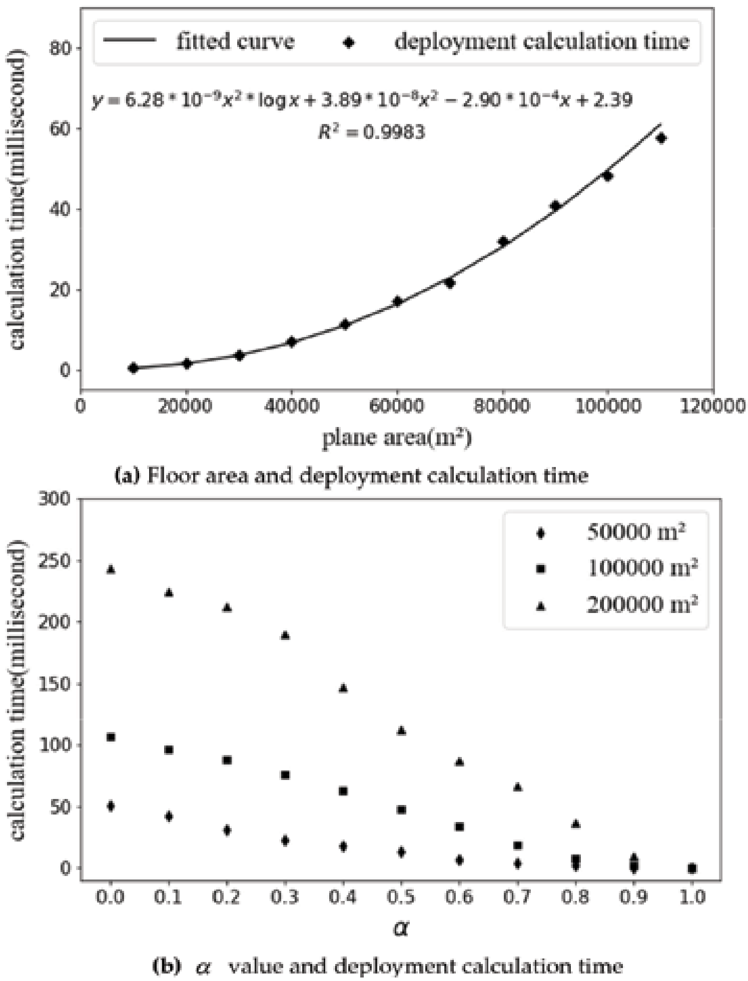


FIGURE 6
Time overhead of deployment algorithm

rithm and α in the case of a fixed space area. The closer α is to 1, the larger the proportion of large space and the larger the coverage of the equilateral triangle.

A large number of AP deployment positions have been colored in the first round, and only a small proportion of vertices need to be iterated using the minimum edge cover and minimum vertex cover algorithms. Thus, the performance is better than that of the space where α is smaller. When α approaches 1, the time complexity of the entire algorithm tends to $O(n)$, and when α approaches 0, the time complexity of the algorithm tends to $O(n^2 \log n)$.

Then, we test the 3-coverage ratio of the signal in the plane space according to the deployment result L. First, we build a test point matrix covering the plane space S with an interval of 1 meter:

$$T = \begin{bmatrix} t_{1,1} & \cdots & t_{1,m} \\ \vdots & \ddots & \vdots \\ t_{n,1} & \cdots & t_{n,m} \end{bmatrix} \tag{4}$$

Then, the set of all test points in the plane space S of the test point matrix is

$$T_s = \{t_{i,j} \mid (i,j) in S\} \tag{5}$$

Normally, 1 meter is also the accuracy limit of the Wi-Fi signal fingerprint map positioning method. For each test point $t_{i,j}$, calculate the number of APs $k_{i,j,L}$ that can be covered by the signal under the L deployment scheme. Count the proportion of measuring points covered by k:

$$p_S(L, T_S) = \frac{|\, t_{i,j} \in T_S \mid k_{i,j,L} \geq k \,|}{T_S} \tag{6}$$

From Figure 7, it can be seen that for the space of $\alpha = 0.5$, the proportion of test points covered by 3 reached more than 95.5%, and the areas not covered by 3 are at the edge of the plane. In addition, most of the areas that have not reached 3 coverage have reached positioning signal 2 coverage. According to the basic principles of the triangulation method, 2-coverage will produce axisymmetric drift, but in most cases, it can also complete more accurate positioning. At the same time, the use of trajectory prediction and other methods can reduce the occurrence of drift.

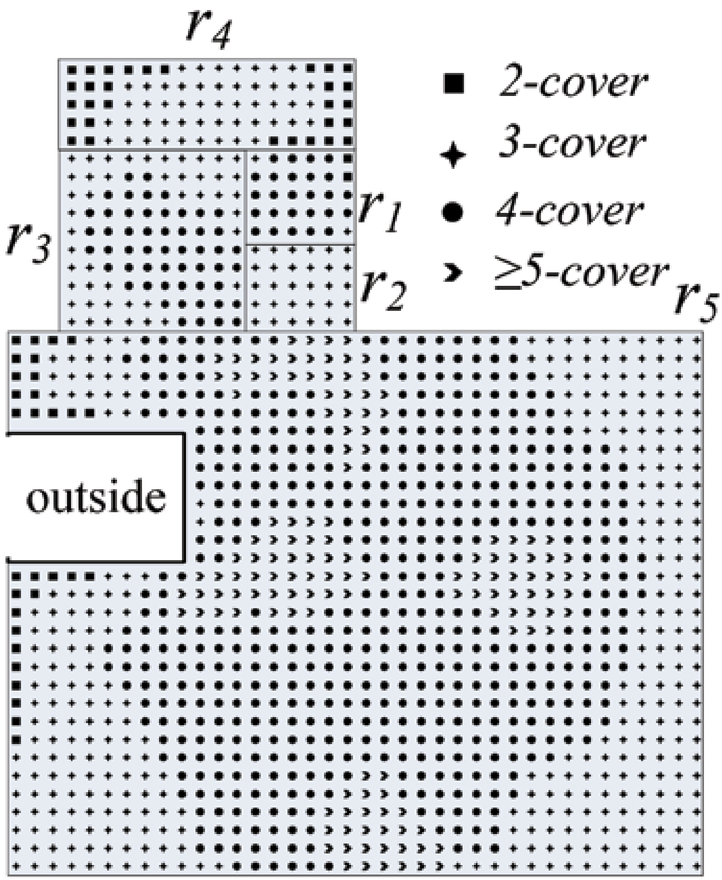


FIGURE 7
Signal coverage rate in plan view

When $\alpha = 0.5$, the average effective coverage density of the signal is k_{avg}. From Figure 8(a), it can be seen that 84.8% of the measurement points are 3-coverage and 4-coverage, and the proportion of measurement points greater than or equal to 5-coverage is 10.77%. Figure 8(b) shows the relationship between α and the average coverage density of the signal. As α increases, k_{avg} decreases slightly and tends toward 3.5. When α approaches 0, Kavg rises but stabilizes at approximately 3.9. The AP deployment scheme obtained by the algorithm in this paper has a signal coverage redundancy within an acceptable range.

It can be seen from the figure that when α approaches 1, the average signal coverage density of the equilateral triangle uniform coverage method is 3.5, which is close to the deployment method in this article; but as α decreases, the average signal coverage density of the equilateral triangle uniform coverage method begins to decrease. When α is close to 0, the average signal

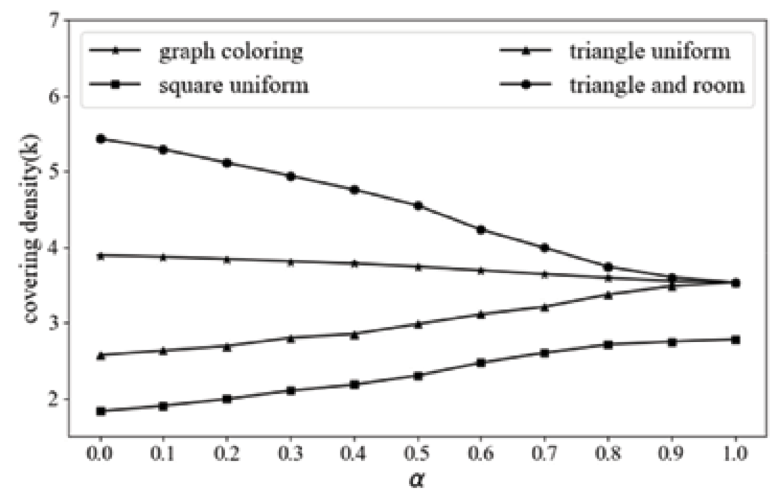


FIGURE 8
Coverage density of deployed algorithm

coverage density of the equilateral triangle uniform coverage method is only 2.57. The square uniform coverage method has a lower average signal coverage.

Figure 8(b) shows that when α is reduced, the proportion of test points whose coverage cannot be achieved by the uniform coverage method increases rapidly. This is due to the attenuation effect of a large number of walls in a small space area, which makes the signal coverage range decrease so that the coverage density is reduced. While the equilateral triangle and room full coverage methods produce a signal coverage density that is too high when α is

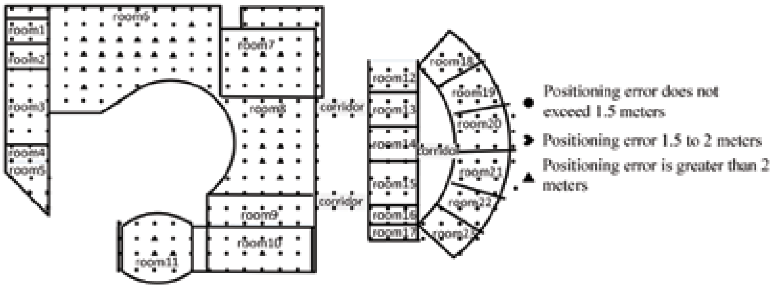


FIGURE 9
Deployment plan and positioning accuracy of Zhejiang Gongshang University Library

small, the density reaches approximately 5.5 when $\alpha = 0$. This greatly increases the deployment and maintenance costs.

This article uses the second floor of Zhejiang Gongshang University Library as an actual case to deploy the AP and test the positioning accuracy. The deployment area has a plane area of 4,200 square meters, including large spaces such as reading rooms and study rooms and office areas dominated by small enclosed spaces. According to the deployment scheme obtained by the deployment algorithm given in this article, we construct the received signal strength (RSS) fingerprint database. First, construct a matrix of reference points with an interval of 2 meters on the second floor of the library, collect signals at the reference points and build a fingerprint map database of the wireless signal strength vector. The positioning algorithm measures the signal strength vector of the current position, and then selects 3 reference points with the shortest distance from the signal strength vector of the current position. Next, the weighted average method is used to calculate the current position.

To prevent the result drift caused by the positioning error, this paper uses the moving window average method to smooth the positioning result, and the window size is 10. The guiding robots of the library needs to obtain the current location information in real time. These devices receive the wireless signal from the APs through the Wi-Fi receiver, and use the positioning algorithm to calculate the specific location, so as to guide the readers. The Movable bookshelves can also obtain the real-time position information of books through this APs. We randomly selected multiple measurement locations to test the positioning accuracy. We can see that the positioning error in most areas is less than 1.5 meters, and the average positioning error is 1.34 meters. The error in the central area and part of the edge area of the large space is also controlled at approximately 2 meters, achieving the high positioning accuracy of Wi-Fi indoor positioning.

more generally, considering the k-coverage of k greater than 3. We only use the regular n-sided shape to cover the plane, where the circumscribed circle diameter of the regular n-sided shape is the coverage radius R of the AP signal. For constructing graph G (V, E) with each corner of positive K-edge as a point, the deployment scheme can also be obtained by graph coloring Deployment algorithm.

6 CONCLUSIONS AND FUTURE WORK

In this paper, we try to solve for the positioning problem of indoor sensor networks by a new AP deployment algorithm. A weighted undirected graph was generated by modeling according to a given indoor plane structure. The basic algorithms of the edge cover set and vertex cover set were used to iteratively color the vertices in the graph, and the AP location deployment scheme was determined in polynomial time. This paper used a simulation to verify and evaluate the computing time of the deployment algorithm and the signal coverage of the deployment scheme. The application was successfully deployed in an actual indoor positioning system, with an average positioning accuracy of 1.34 meters. Practice showed that building an indoor positioning system based on the algorithm described in this article has lower cost and higher accuracy and is suitable for the deployment of sensor network positioning systems required for automated mobile devices.

In the future we will study the AP deployment method in irregular indoor space, such as multi-floor and semi-open space. The coverage of signals in these spaces needs to be modeled and solved by heuristic algorithm.

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