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Critical Path for Projects with Activity Durations Modelled as Z-Fuzzy Numbers

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The objective of the research is to test the method of critical path calculation with fuzzy activity durations in different possible scenarios and to suggest method's improvements. The method uses Z-fuzzy numbers and incorporates historical records to assess credibility. An experiment with sequential activities was constructed. Normal distribution was used to reflect the behavior of human estimators. The scenarios were generated using computer simulations. The error in the critical path length was calculated. Two basic adjustments of the method have been identified (adjusted formula, unified conversion rule). The results show that there is a gain when using the adjusted method, which is observed for pessimists or optimists or for the volatile case, but only if it contains a predominance of optimistic (or pessimistic) estimations. The adjusted method can be used within the teams that experience a bias in their estimations, e.g. because of the customer that has strict time expectations.

Keywords: Critical path, activity durations, Z-fuzzy numbers, project management, credibility assessment, optimism bias

1 INTRODUCTION

In our previous paper [1] a method for determining a critical path of a project with fuzzy activity durations (based on Z-fuzzy numbers that include credibility assessment of experts) was presented. It has to be emphasized that we conducted a literature search in Mendeley, in Scopus and ScienceDirect and we did not identify any additional works referring to critical path in the context of Z-fuzzy numbers. At the same time, other types of fuzzy numbers

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(hexagonal, octagonal, type-2, hesitant, intuitionistic ([2] - [12]) have been used to model project networks and to identify the critical path. Pythagorean fuzzy numbers for project critical path problem are used in [13] and, additionally, the work uses experts weighting method. Type-2 fuzzy numbers are used in [14] and in [15], which also includes subjective experts judgements, as well as in [16], where they are utilized to represent uncertainties. The general properties and applications of intuitionistic, pythagorean, type-2 and other fuzzy numbers (like fermatean) can also be found in: ([17] – [21])

Having in mind that Z-fuzzy numbers represent a new perspective in fuzzy modelling, allowing to model experts' credibility, we believe that [1] and the present paper cover an important research gap in project time management. Z-fuzzy numbers are constructs composed of two elements: a "classical" or ordinary fuzzy number (e.g. a triangular, trapezoidal. L-R, etc.) given by an expert and an evaluation of the experts' credibility. The second term is then used to transform the first term (reflecting the original expert's opinion) to a "classical" fuzzy number. It represents the adjusted expert's opinion (using his or her credibility as the adjustment basis).

Another gap identified in the literature concerns Z-fuzzy numbers themselves. Their identified applications to project management ([22] - [24]), in other contexts than the critical path, are all based on the assumption that the expert's lack credibility means the need to decrease their original evaluation. The known methods of transforming Z-fuzzy numbers into "classical" fuzzy numbers are based on this assumption. This may be an adequate approach also in case of project time management if experts can be considered as pessimists, so that the estimation of activities duration provided by them should be reduced in order to make them realistic. However, when experts are asked to estimate project activities duration, their lack of credibility may express itself in different ways. According to [25], we can distinguish three groups of project time estimators: optimists, pessimists, and volatile experts. Literature shows that all those cases may occur in practice, although the intensity of each estimator type occurrence may depend on the project type and on project team features. For example, Goldratt [26], the author of the critical chain concept (originally in the IT domain), claims that most people, if not motivated to do otherwise, give pessimistic evaluations of activities duration times. On the other hand, there are numerous reports from practice indicating the opposite tendency, thus the optimism bias [27], especially in teams [28]. We decided to use the most general approach from [25].

Based on the estimators' classification from [25], in [1] we used a modified definition of Z-number and a novel method of transforming a Z-number into an ordinary fuzzy number. The historical data were used to determine the estimation bias in the past and the features of each human estimator (whether they were pessimistic, optimistic, or volatile in the past). Additionally, the experts without any estimation bias were also distinguished. For this fourth type of experts no adjustments were needed. Using available historical data, *Z*-fuzzy numbers corresponding to the experts' estimations were determined. The second terms of the numbers represented the estimation bias. As the last step of the method, the Z-numbers were converted back to ordinary fuzzy numbers that constituted the input to the fuzzy critical path methods already known in the literature. Any of the known fuzzy critical methods based on estimations of activities duration in the form of "classical" fuzzy numbers, known from the literature, can be used here (e.g. [6], [29] – [33]).

There are two basic problems linked to the application of Z-numbers to project time management. First concerns the identification of the estimation bias or estimation credibility. The second problem is linked to the specific formula which should be used to transform the Z-number to an ordinary fuzzy number. Paper [1] contains a proposal of answering these problems, but this proposal has not been verified in practice. The objective of the research presented in this paper is to test the method of [1] in different possible scenarios, along with a modification proposal, and to arrive at practical implications for the practice of project time management.

As computer simulation was used for the verification of both proposed versions of the method, several additional assumptions were made to achieve the stated objectives. It was assumed that the degree of optimism and pessimism can be expressed by a probability distribution. Moreover, it was assumed that optimism and pessimism degrees are expressed by normal distributions. In the research a relatively simple project structure was used, so that it was easy to observe the influence of optimism and pessimism of human estimators on the resulting critical path.

The outline of the paper is as follows: Section 2 presents the definitions and notations needed as well as it introduces the original method that is subject to the research conducted in this paper. It also presents the method modifications that were suggested due to the research. Section 3 presents the experiment that was used to test the method and to identify possible method's adjustments. It contains the list of assumptions used for the experiment as well as introduces scenarios that were tested. Section 4 presents experimental results. The results of the original method are presented first, and they are further compared with the results of the modified (adjusted) method. Section 5 discusses the results obtained in the broader context of existing methods and use cases. It also tries to indicate future research directions.

2 ORIGINAL METHOD

2.1 Basic definitions and notations from the literature

A Z-fuzzy number or a Z-number [34] is a couple $\left(\tilde{A}, \tilde{Z}\right)$, where $\tilde{A}$ and $\tilde{Z}$ are ordinary fuzzy numbers, and the support of $\tilde{Z}$, according to the original definition of Z-numbers, is included in the interval [0,1]. $\tilde{Z}$ represents the credibility of the expert opinion $\tilde{A}$ or the possibility that evaluation $\tilde{A}$ is correct.

The closer to 1 the support of $\tilde{Z}$ is, the higher the credibility of $\tilde{A}$. In the literature, a Z-fuzzy number $\left(\tilde{A},\tilde{Z}\right)$ is eventually expressed as a classical fuzzy number $\tilde{B}$. The following approach is usually used [35]:

$$\tilde{B} = d\left(\tilde{Z}\right)\tilde{A} \tag{1}$$

where $d\left(\tilde{Z}\right)$ is a crisp defuzzification of fuzzy number $\tilde{Z}$.

$d\left(\tilde{Z}\right)$ can be the square root of the fuzzy centroid of $\tilde{Z}$ [22] or any other crisp representation of the fuzzy number $\tilde{Z}$, see e.g. [36] and [37].

In the above approach credibility lower than 1 has to lead to the adjustment of the first term of the Z-number ($\tilde{A}$) in the sense of a shift of its support and core value to the left.

In this paper the ordinary fuzzy numbers will be triangular fuzzy numbers, represented by their minimal, core and maximal values. For example, $\tilde{A}$ will be represented as $\left(\tilde{A}_a,\tilde{A}_b,\tilde{A}_c\right)$.

2.2 Original method description

The method presented in [1] for determining a critical path of a project using *Z*-fuzzy numbers that is under further consideration in this paper is an extension of the classical fuzzy CPM approaches known in the literature. In the classical fuzzy approaches to the critical path problem (those based on triangular fuzzy numbers) the decision makers (usually persons responsible for individual activities) asses the duration of project activities in the form of a triple: optimistic, average (most possible) and pessimistic duration (left hand upper part of Figure 1). Then the respective fuzzy critical path method is applied (the bottom part of Figure 1, the fuzzy CPM methods may be e.g., those from [6] and [29] – [33].

The extension, proposed in [1], was composed of two steps:

- a credibility assessment step (right hand upper part of Figure 1) - that completed the evaluation performed in the left-hand upper part of Figure 1 by the assessment of its (or rather of its author) credibility (in Figure 1 denoted as "Fuzzy credibility"), and
- *Z*-fuzzy to fuzzy conversion step that allows to convert the obtained *Z*-fuzzy numbers back to classical triangular fuzzy numbers, which represent the original assessment of durations adjusted on the basis of the credibility of the decision maker (estimator).

In the considered method the credibility assessment is based on historical records of estimated and actual duration times. The records concerning similar projects and the same decision makers should be taken into account. This step, whose objective is the determination of the second term of Z-numbers,

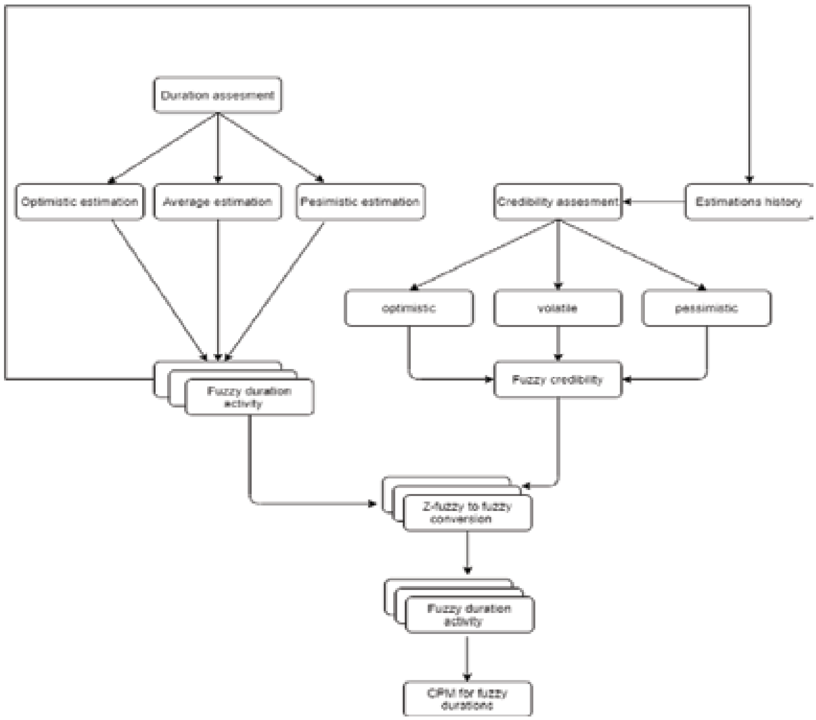

FIGURE 1
CPM method that includes fuzzy activity durations and experts' credibility assessments (represented as *Z*-fuzzy numbers) as presented in [1]

is based on calculating the deviations between real (actual) values and estimated values in the past. For each activity we can use the same historical record or a different one, depending on the persons involved in the estimations and the activity type.

Thus, the joint result of the left hand and right hand upper parts of Figure 1 will be, for each project activity, a Z-number $\left(\tilde{A},\tilde{Z}\right)$, where $\tilde{A}=\left(\tilde{A}_a,\tilde{A}_b,\tilde{A}_c\right)$ is the possibly biased estimation given directly by the estimator in the left hand upper part of Figure 1, and $\tilde{Z}=\left(\tilde{Z}_a,\tilde{Z}_b,\tilde{Z}_c\right)$, determined by the procedure denoted as "Credibility assessment", is determined as follows

$$\tilde{Z}=\left(\tilde{Z}_a,\tilde{Z}_b,\tilde{Z}_c\right)=\left(\begin{array}{l} 1+\dfrac{\sum_{j=1}^{N}\left(R_a^j-\tilde{A}_a^j\right))/\tilde{A}_a^j}{N},1+\dfrac{\sum_{j=1}^{N}\left(R_b^j-\tilde{A}_b^j\right))/\tilde{A}_b^j}{N},1 \\ +\dfrac{\sum_{j=1}^{N}\left(R_c^j-\tilde{A}_c^j\right))/\tilde{A}_c^j}{N} \end{array}\right) \tag{2}$$

where:

$\tilde{Z}=\left(\tilde{Z}_a,\tilde{Z}_b,\tilde{Z}_c\right)$ - fuzzy number representing the credibility of past estimations

$\tilde{A}_j=\left(\tilde{A}_a^j,\tilde{A}_b^j,\tilde{A}_c^j\right)$ - estimated value in the j-th record, j=1,…,N

$\tilde{R}_j=\left(\tilde{R}_a^j,\tilde{R}_b^j,\tilde{R}_c^j\right)$ - real (actual) value in the j-th record, j=1,…,N

N - number of historical records taken into account.

The illustration of the Z-fuzzy numbers used in this article is presented in Figure 2, where both numbers are triangular fuzzy numbers. The actual values $\tilde{R}_j=\left(\tilde{R}_a^j,\tilde{R}_b^j,\tilde{R}_c^j\right)$ will usually be crisp, but we allow here the possibility of the lack of full knowledge about the actual task duration (for example in case of research activities, which are difficult to measure because of the inaccurate knowledge about the duration of creative processes)

Number $\tilde{Z}=\left(\tilde{Z}_a,\tilde{Z}_b,\tilde{Z}_c\right)$ represents the general average credibility of past estimations. As we can see, in our approach the support of the second term of the Z-number does not have to be included in the interval [0,1]. Values over 1 are also possible. This is the consequence of the fact that we allow the adjustment not only to reduce the original estimation $\tilde{A}$=$\left(\tilde{A}_a,\tilde{A}_b,\tilde{A}_c\right)$ (thus shift its support and core to the left), but also to increase it or apply both a reduction and an increase to different parts of its support.

Before Z-number $\left(\tilde{A},\tilde{Z}\right)$ is converted to a triangular fuzzy number, another step is carried out for each expert's estimation that is aimed at determining expert type. Expert type determination is based on past expert estimation performance and realized with the usage of the rules from Table 1, performed on the historical data records for the given expert. In the method of [1] it is assumed that the expert type is obtained as the most common label occurring among all historical records of the given expert. For example, if the label

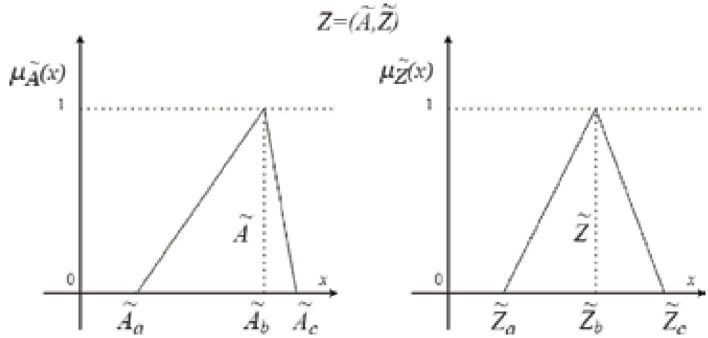

FIGURE 2
Illustration of Z-fuzzy numbers used in this article (both $\tilde{A}$ and $\tilde{Z}$ numbers are triangular fuzzy numbers, $\tilde{A}$ is a real-valued uncertain estimation, $\tilde{Z}$ represents credibility)

"optimistic" occurred most often in the records of the given expert, this expert will be further considered as optimist. In case of ties, an external decision maker needs to decide which category should be assigned to the given expert.

Table 2 allows to express the biased estimations given by experts, where the bias is expressed by the second term of Z-numbers, by means of adjusted classical triangular fuzzy numbers. In [1] we assumed that this conversion should depend on the expert type.

Table 2 corresponds to the box 'Z-fuzzy to fuzzy conversion' in Figure 1. In case of an optimist the maximum of the support of $\tilde{Z}$ (which in case of optimist will be higher than 1) is used for the multiplication: an optimist is bound to have given significantly too short duration times, so they need to be strongly increased. The pessimist's estimations should be strongly reduced, thus the (significantly smaller than one) minimum of the support of $\tilde{Z}$ is used. The volatile expert's estimations are adjusted using all the three basic characteristics of fuzzy number $\tilde{Z}$.

In this stage any of the known fuzzy critical path determination methods (based on triangular fuzzy numbers) can be applied to the project network - where the activities are assigned duration times $\tilde{B}$. The critical path(s) generated in this way represent(s) the solution searched for.

Rule no.	Condition	Label applied
(1)	$\tilde{R}_a^j - \tilde{A}_a^j > 0$ AND $\tilde{R}_b^j - \tilde{A}_b^j > 0$ AND $\tilde{R}_c^j - \tilde{A}_c^j > 0$	"optimistic" label
(2)	$\tilde{R}_a^j - \tilde{A}_a^j < 0$ AND $\tilde{R}_b^j - \tilde{A}_b^j < 0$ AND $\tilde{R}_c^j - \tilde{A}_c^j < 0$	"pessimistic" label
(3)	otherwise	"volatile" label

TABLE 1
Rules used for obtaining expert type

Rule no.	Expert feature	Conversion applied
(4)	"optimistic"	$\tilde{B} = (\tilde{B}_a, \tilde{B}_b, \tilde{B}_c) = (\tilde{A}_a\tilde{Z}_c, \tilde{A}_b\tilde{Z}_c, \tilde{A}_c\tilde{Z}_c)$
(5)	"pessimistic"	$\tilde{B} = (\tilde{B}_a, \tilde{B}_b, \tilde{B}_c) = (\tilde{A}_a\tilde{Z}_a, \tilde{A}_b\tilde{Z}_a, \tilde{A}_c\tilde{Z}_a)$
(6)	"volatile"	$\tilde{B} = (\tilde{B}_a, \tilde{B}_b, \tilde{B}_c) = (\tilde{A}_a\tilde{Z}_a, \tilde{A}_b\tilde{Z}_b, \tilde{A}_c\tilde{Z}_c)$

TABLE 2
Rules used for Z-fuzzy number $(\tilde{A}, \tilde{Z})$ to standard triangular fuzzy number $\tilde{B}$ conversion

2.3 Modification of the method

As we mentioned in the introduction, there are two main problems linked to Z-numbers: the determination of credibility and the conversion of a Z-number to an ordinary fuzzy number. Both steps have to be performed in such a way that the results reflect the reality, thus the actual influence of the estimator personal features on the estimations he/she provides, to the highest possible degree. In order to achieve this, it is necessary to verify various formulae in practice. In this paper, we tested the approach determined by formula (2) (credibility assessment) and Table 2 (conversion to ordinary fuzzy numbers) from [1], but we also tested a novel approach, given by formula (3) (credibility assessment) and Table 3 (conversion to ordinary fuzzy numbers).

$$\tilde{Z} = \left(\tilde{Z}_a, \tilde{Z}_b, \tilde{Z}_c\right) = \left(\frac{1}{1 - \frac{\sum_{j=1}^{N} \left(R_a^j - \tilde{A}_a^j\right) / \tilde{A}_a^j}{N}}, \frac{1}{1 - \frac{\sum_{j=1}^{N} \left(R_b^j - \tilde{A}_b^j\right) / \tilde{A}_b^j}{N}}, \frac{1}{1 - \frac{\sum_{j=1}^{N} \left(R_c^j - \tilde{A}_c^j\right) / \tilde{A}_c^j}{N}} \right) \quad (3)$$

Formula (3) represents another method to express by a number greater than 1 the situation when the actual values usually exceed the planned ones and by a number smaller than 1 the opposite situation. The objective to test the approach from Table 3 arose from the intention to verify whether the introduction in [1] of estimator types was meaningful.

3 COMPUTATIONAL SIMULATION

As it was not yet possible to obtain real-world estimations history for the purpose of tests, the research method that was used to test the method – proposed in [1] and modified in this paper - was simulation.

Rule no.	Conversion applied to all expert types
(7)	$\tilde{B} = \left(\tilde{B}_a, \tilde{B}_b, \tilde{B}_c\right) = \left(\tilde{A}_a\tilde{Z}_a, \tilde{A}_b\tilde{Z}_b, \tilde{A}_c\tilde{Z}_c\right)$

TABLE 3
Rules used for Z-fuzzy number $\left(\tilde{A}, \tilde{Z}\right)$ to standard triangular fuzzy number $\tilde{B}$ conversion – modified

The experiment to test the method that was conducted in the current paper was composed of the following steps:

(a) sample project structure was chosen for all calculations
(b) fixed set of normal distribution parameters range to be tested was chosen and three types of experts were modelled using these parameters
(c) number of historical records available was chosen as another parameter to be tested and different scenarios with different numbers of historical records available were used to estimate *Z*-fuzzy numbers
(d) number of biased estimations vs number of unbiased estimations was also parametrized
(e) sample data records were generated using simulations
(f) validation data were generated, and the measures were calculated on this data, so that it was possible to verify the appropriateness of the method.

3a Sample project structure

A project structure that was used for the experiment is presented in Figure 3. It can be observed that the crisp critical path has duration 90 (duration 10 times 9).

Activities *A1* … *A9* were used to simulate the biased experts' estimations.

3b Normal distribution parameters

The parameters of the experiment were used for generating normal distributions. Both means and standard deviations were measured in percentage units. The set of mean values that were taken represented relative shifts from real values. The set of standard deviations taken (3σ) represented the spread of possible values. For example, when $(\mu, 3\sigma)$ was taken, almost all the simulation results should be within the $[-3\sigma; +3\sigma]$ interval, which in this paper takes the form of $[R + (\mu R - 3\sigma R)/100; R + (\mu R + 3\sigma R)/100]$ and most of the values should oscillate around μ, which here takes the form of $R + \mu R/100$, where R is a hypothetical real value that should be estimated, when experts don't have any bias (of optimistic, pessimistic or volatile behavior). For example, when the hypothetical real value 10 is considered and the expert is a 10% optimist ($\mu = -10\%$) with a 50% spread ($3\sigma = 50\%$), then almost all the simulation

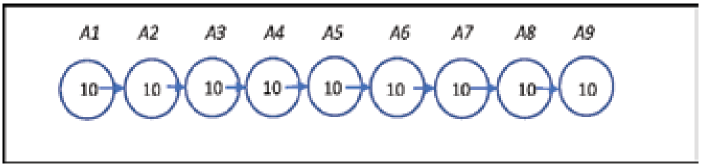

FIGURE 3
Project structure used for the experiment. Symbols *A1*, …, *AN* above the circles represent activities. Numbers inside the circles represent crisp activity durations

results should be within the interval of [R + (-10%R – 50%R) / 100; R + (-10% R +50%R) / 100] = [10 + (-100 – 500)/100; 10 + (-100 + 500)/100] = [4; 14] and most of the values should oscillate around R + μR/100 = 10 - 100/100 = 9.

The following set of parameters was used depending on the scenario (Table 4).

Means are fixed for pessimistic and optimistic scenarios and are randomly selected for volatile scenario. Standard deviation is fixed for all scenarios (Figure 4).

Different parameters were assigned to the activities *A1* … *A9* using the following key (Table 5).

	Tested set of μ values	**Tested set of 3σ values**
Pessimistic scenario	{10%, 30%, 50%}	{10%, 30%, 50%}
Optimistic scenario	{-10%, -30%, -50%}	{10%, 30%, 50%}
Volatile scenario	{random[-10%; 10%], random[-30%; 30%], random[-50%;50%]}	{10%, 30%, 50%}

TABLE 4
Normal distribution parameters used for the experiment

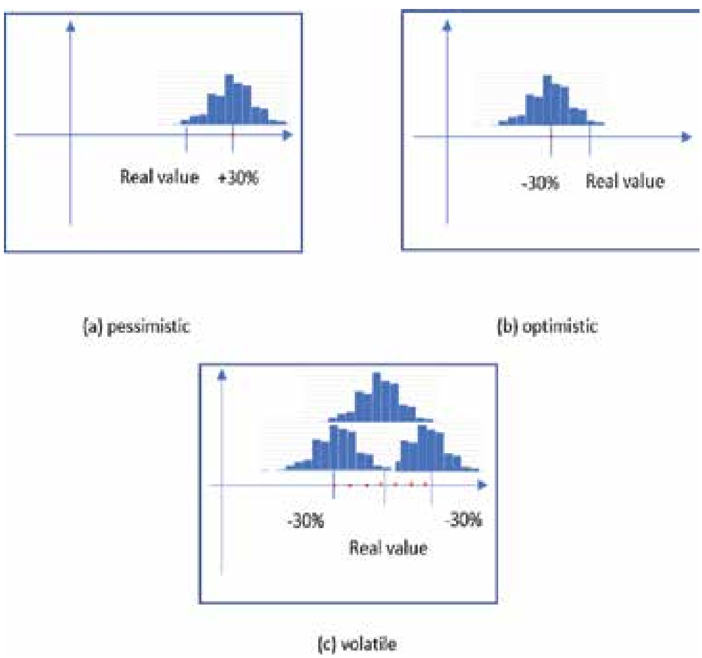

FIGURE 4
Visual representation for pessimistic (a), optimistic (b) and volatile (c) scenarios. Red dots represent sample means taken depending on the scenario

	A1	*A2*	*A3*	*A4*	*A5*	*A6*	*A7*	*A8*	*A9*
Hypothetic real value	*10*	*10*	*10*	*10*	*10*	*10*	*10*	*10*	*10*
	$(\mu, 3\sigma)$								
Pessimistic scenario	(10%, 10%)	(30%, 10%)	(50%, 10%)	(10%, 30%)	(30%, 30%)	(50%, 30%)	(10%, 50%)	(30%, 50%)	(50%, 50%)
Optimistic scenario	(-10%, 10%)	(-30%, 10%)	(-50%, 10%)	(-10%, 30%)	(-30%, 30%)	(-50%, 30%)	(-10%, 50%)	(-30%, 50%)	(-50%, 50%)
Volatile scenario	(random [-10%; 10%], 10%)	(random [-30%; 30%], 10%)	(random [-50%; 50%], 10%)	(random [-10%; 10%], 30%)	(random [-30%; 30%], 30%)	(random [-50%; 50%], 30%)	(random [-10%; 10%], 50%)	(random [-30%; 30%], 50%)	(random [-50%; 50%], 50%)

TABLE 5
Parameters assigned to the activities

As it can be observed, the standard deviation used for the experiment grows with the activity index. Standard deviation (3σ) is equal to 10% for the first three activities, then it is equal to 30% for the next three activities and then it equals 50% for the last three activities.

The simulation that was used was designed using the following idea: providing that real values for all nine activities should equal ten (arbitrary units, for example days), the random bias was generated from distributions given above, causing the estimated values to be different from real values. Then the *Z*-fuzzy values were calculated. At the end, completely new dataset was generated using the same distribution parameters, the *Z*-fuzzy to fuzzy number conversions (proposed in Tables 2 and 3) were applied to this new data records and the critical path length was determined using the simplest possible standard approach (i.e., conversion from the fuzzy number to a crisp number based on the formula $\left(\left(a+2b+c\right)/4\right)$) and applying the standard critical path method). The resulting critical path length was compared to the desired critical path length of 90 and compared to the critical path calculated using not the adjusted *Z*-fuzzy numbers-based estimations, but the unadjusted original fuzzy estimates. The final errors were calculated as percentage values (showing how much the obtained result differs from the desired result). The method requires that each expert gives one's estimation by providing three fuzzy values: minimum, average, and maximum. For this reason, for each expert estimation simulation the three values were generated from a given distribution and the values were sorted in ascending order, producing simulated estimations for minimum, maximum and average.

3c Number of historical records

It was assumed that the method can have different numbers of historical records available or taken into account. The number of records considered is denoted here as *N*. The greater *N*, the more historical records are considered, so the more accurate results should be obtained. On the other hand, the "profile" of an expert may evolve over time, which means that the value of *N* cannot be too big, because only recently estimated values are in such a case important to be considered. For this research the following values of *N* were tested: *N = {5, 10, 30, 50, 100}*. Additionally, the case when no adjustment of original estimations was performed (so that $N = 0$, because no historical records were used) was also tested.

3d Number of biased estimations

The number of biased estimations is important, because the more biased experts are considered, the less accurate the final critical path should be, and more corrections are needed in such a case. For the purpose of this research, a relatively simple scenario was tested, where the number of biased estimations grew from one to nine with the increment of three for the corresponding activities. This model also incorporates the growth of the spread of experts' estimations (standard deviation). The first three experts had the spread of 10%, the next three had the spread of 30% and the last three the spread of 50%. The scenarios tested are then as follows: *M = {3, 6, 9}*, where *M* is the number of biased estimations. The number of unbiased estimations would be then equal *9-M*.

3e Sample data records

Summing up, the parameters considered in subsections presented above produced the sample data records and included the following scenarios (Table 6), where *Q*(*i,j*) stands for quality measure that was used to test the parameters. Additionally, a row *No credibility assessment was used* was inserted to show the quality when no credibility assessment (and no *Z*-fuzzy numbers) would be applied – the original experts' estimations would stay unchanged and would be directly taken for calculating the critical path.

Because for optimistic and pessimistic simulations analogous corrections are used, for the purpose of this research only optimistic and volatile scenarios were tested. For pessimistic scenario, the results should be similar to the results for optimistic ones and will not be presented.

Optimistic Simulations / Volatile simulations	***M=3***	***M=6***	***M=9***
No credibility assessment was used	*Q(0,3)*	*Q(0,6)*	*Q(0,9)*
N=5	*Q(5,3)*	*Q(5,6)*	*Q(5,9)*
N=10	*Q(10,3)*	*Q(10,6)*	*Q(5,9)*
N=30	*Q(30,3)*	*Q(30,6)*	*Q(10,9)*
N=50	*Q(50,3)*	*Q(50,6)*	*Q(50,9)*
N=100	*Q(100,3)*	*Q(100,6)*	*Q(100,9)*

TABLE 6
Tested scenarios summary for optimistic case and volatile case

3f Validation data records

As it was mentioned above, the validation data records were generated from the same distribution as in case of sample data records. The difference is that the simulation was performed more times (*500*) and the mean value for each scenario was obtained. The error measure was applied to this mean value as the difference (in percent) between the desired critical path length (of *90*) and the obtained critical path length.

4 RESULTS FOR THE ORIGINAL METHOD

The results for the originally suggested method of [1] will be presented here. These are the results without any modifications with respect to the original method from [1] yet, besides one. This single adjustment is that the real values obtained from the dataset of historical values should be described using only one value R_j and not using the three values $\left(\tilde{R}_a^j, \tilde{R}_b^j, \tilde{R}_c^j\right)$. It is because the real values are not fuzzy and there is no need to represent them using fuzzy numbers. The opposite case (real values being fuzzy) would be considered if the measurement of the real values was imprecise. For activity durations considered in this paper it is not the case (i.e., final real-value measurements are crisp). Let us start with the optimistic scenario.

For simulations we used Microsoft Excel. It allows to easy generate random numbers from normal distribution (Figure 5).

The estimation of Z was also done in Microsoft Excel (Figure 6). Each step (or sub-step) of the simulation had its own tab.

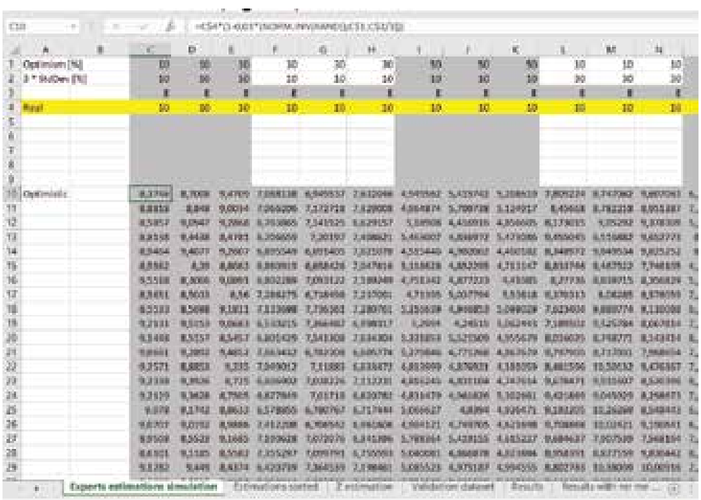

FIGURE 5
Optimistic estimations simulation

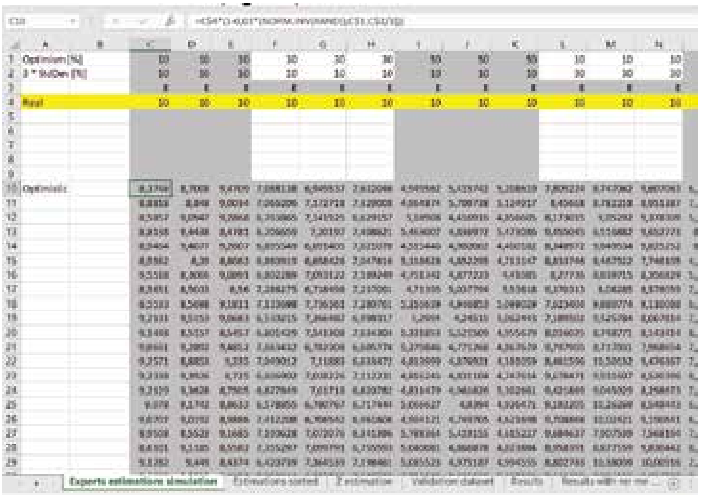

FIGURE 6
Z-estimation, based on simulated values

Error [%]	***M=3***	***M=6***	***M=9***
No credibility assessment was used	10.03	19.98	30.04
N=5	1.52	4.41	**10.41**
N=10	1.37	**4.27**	11.21
N=30	1.30	4.64	11.84
N=50	**1.22**	4.67	12.03
N=100	1.33	4.86	12.36

TABLE 7
Results for the original method [1] in optimistic scenario

Due to the limited size of this article, we presented above only a few sample Microsoft Excel formulas that were used in the experiment.

4a Original method results for the optimistic scenario

It can be observed that the results of the method produce the greater error, the more bias the expert estimations have. It can also be observed that without applying the method (and directly using experts' estimations without considering their credibility), the results would be much worse and could produce even 30% of error, due to the experts' bias (Table 7). The best results for a given *M* is bold.

4b Original method results for the volatile scenario

The scenario presented in Table 8 was generated for the experts that are volatile, i.e., sometimes they act like optimists and sometimes like pessimists. For this

Error [%]	*M=3*	*M=6*	*M=9*
No credibility assessment was used	**0.08**	**0.17**	**0.23**
N=5	1.63	2.14	3.94
N=10	1.77	2.82	4.39
N=30	0.67	1.30	1.88
N=50	0.83	1.82	2.58
N=100	0.51	1.54	2.25

TABLE 8
Results for the original method [1] in volatile scenario (when experts are equally probable to be optimists or pessimists)

specific scenario the probability of acting as an optimist and acting as a pessimist is equal and equals 0.5. It can be observed that the results for the volatile scenario are, unexpectedly, much better than the results for the optimistic scenario. What is also very interesting is that the results without applying credibility assessment are much better then when including credibility and contain very small error. This could indicate that the method should probably not be applied in such a scenario, because it can worsen the expected results. This can also indicate that when experts act sometimes as optimists and sometimes as pessimists and the number of optimistic records is roughly the same as the number of pessimistic records (the probabilities of being an optimist and the probability of being a pessimist are equal = 0.5), the optimistic and pessimistic behavior can counteract and the product of such a behavior is like the behavior of non-biased experts.

The results above show that there is a gain when applying credibility assessment in case of the optimistic or pessimistic bias, but the method seems to be ineffective when applied to the volatile experts who sometimes act as pessimists and sometimes act as optimists, so in fact they are non-biased experts (on average).

As next step, we applied the modification from section 2.3, in order to test another method for credibility assessment and to verify whether the distinction of various estimator types was important.

4c Modified method results

In this part of the simulation, we used formula (3) for credibility assessment and Table 3 for Z-fuzzy to ordinal fuzzy number conversion. The experimental results showed that in case of the optimists, the modification proposed in section 2.3 produces the best results. They are presented in Table 9.

When applying the method after adjustments to the volatile case of experts being equally optimists and pessimists, the results are similar to the results of the method originally presented in [1], so they show that the method should probably not be applied as it can worsen the results (Table 10).

Error [%]	***M=3***	***M=6***	***M=9***
No credibility assessment was used	10.01	19.95	29.95
N=5	0.40	1.85	1.19
N=10	0.17	0.85	1.51
N=30	0.06	0.80	**0.32**
N=50	**0.01**	0.55	**0.32**
N=100	0.04	**0.23**	0.41

TABLE 9
Results for the adjusted method in optimistic scenario

Error [%]	***M=3***	***M=6***	***M=9***
No credibility assessment was used	0.17	**0.08**	**0.16**
N=5	0.39	0.94	2.30
N=10	0.67	1.27	2.41
N=30	0.79	1.24	1.24
N=50	0.33	0.72	0.80
N=100	**0.02**	0.36	0.61

TABLE 10
Results for the adjusted method in volatile scenario (when experts are equally probable to be optimists or pessimists)

Error [%]	***M=3***	***M=6***	***M=9***
No credibility assessment was used	2.12	4.12	6.15
N=5	0.92	2.61	2.55
N=10	0.47	1.60	1.69
N=30	**0.21**	0.33	**0.03**
N=50	0.38	0.06	0.23
N=100	0.27	**0.04**	0.22

TABLE 11
Results for the adjusted method in volatile scenario (when experts are more probable to be optimists, with a probability of 0.7)

It is worth considering the case when experts are volatile and are not equally pessimists or optimists, but they are shifted towards one of the two, e.g., towards being an optimist (with a probability of 0.7). It turns out that the method produces much better results in this case (Table 11).

5 DISCUSSION AND FURTHER RESEARCH

The experiment constructed to test the method was designed to initially check the method from [1] and its modification proposed here. It was constructed as a hypothetical case when experts being optimists or pessimists tend to regularly underestimate or overestimate according to the bias that can be described using normal distribution. Also, the volatile case has been tested to include the behavior of experts who sometimes act as optimists and sometimes as pessimist and was modelled as a combination of the two.

It has been shown by the experiment that the application of Z-numbers to project time management can bring better results than using original fuzzy estimations, generated directly by the persons responsible for the execution of individual activities. However, it did not happen in all the cases. The method achieved better results for optimists (or pessimists), but not for volatile experts. However, when the volatile case has been modified to model experts who are indeed both optimists and pessimists, but they tend to be more optimists, the bias has also significantly been reduced by the method. We also showed that the modification proposed in this method leads to better results in selected cases.

The original fuzzy estimations stem from human beings who are exposed to various external factors and have individual personality features that must influence somehow the estimations of activities duration times provided by them. It needs to play a role whether the estimator is afraid or not of being held accountable in the future for not keeping to the estimated time (this is an important justification element of the critical chain method [26]), and this is influenced by experience of the estimator and his or her personal features. There is a strong influence of the team on the individual estimations: if the team offers support and friendly atmosphere, the individual members will be readier to make optimistic estimations. Finally, someone who feels more familiar with the methods or technology that will be used in the respective activity will be more likely to give optimistic duration estimations, even if in reality this familiarity with the methodology does not have substantial influence on the actual duration.

The experiment has also shown that the choice of credibility assessment and the method of adjusting the original estimation influences the quality of the results. In our paper we based the two aspects on historical records from similar projects that show the differences between past estimations the actual values. Such an approach can be used only if the organisation has relevant data at their disposal. If not, other methods have to be applied. In fact, they should be applied even if sufficiently comprehensive historical records are available. For example, factors influencing estimations considered in the COCOMO methods ([38] and [39]) should be taken into account, which concern the team, the customer and the relations of the estimator with them, as well as the expertise and experience of the estimator in the given field. Even if some of these aspects are soft and seem difficult to measure, they still can be measured to a certain extent. In the literature there are numerous proposals of questionnaires allowing to measure soft aspects like emotional intelligence, teamwork, leadership impact, etc. (e.g. [40] and [41]), let alone the much more measurable aspects, like competences and experience. Such questionnaires may provide quantitative data, allowing a reliable generation of a Z-fuzzy project activity estimation and its adjusted ordinary fuzzy form.

As the research presented in this paper is not based on real world data records, it doesn't allow to formulate a final judgment of the performance of the method. However, it is the first step that permits to formulate the hypothesis that the method may be of practical importance. Collecting appropriate number of real-world historical data and applying the method to real world projects would be a natural extension of the approach presented in this paper and is planned as future research.

In the future, the usage of other types of Z-numbers can be considered, in which not only the membership, but also non-membership and hesitancy degrees can be taken into account (intuitionistic, pythagorean or spherical numbers). However, the introduction of more complex Z-fuzzy numbers has to be preceded by a positive validation of the simpler approach proposed here. In accordance with the modelling principle called ASANA (As Sophisticated As is Necessary to Assure) [42], in project management we should not strive at any cost at obtaining complex formal constructs. They might not add much value to the efficiency and efficacy of the project management process, and the price of their practical application (in terms of human effort and stress needed) may be unacceptable. This price has a chance to become justifiable only once simpler approaches have been accepted by project managers in practice. Thus, first steps in further research should be case studies where the approach considered here would be tested and their acceptance measured.

Also, other aspects of project time estimation should be considered in future research. For example, may find out that human optimism or pessimism can evolve over time. For this reason, the choice of historical records that should be used is of a great importance. Reaching with the records too far to the past would probably not improve the performance of project time estimation and could even worsen the results, as it would include estimations that might be out of date. This aspect and the factors influencing estimations mentioned above will be the subject of future research.

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